EE 567

Homework 5

Due Tuesday, October 1, 2019

Work all 4 problems.

Problem 1. Consider a first-order PLL as described in class, i.e., the loop filter is $H(f) = 1$. Then

$$\Phi_e(f) = \frac{1}{1 + K_0/jf}\Phi_1(f).$$

We wish to investigate the loop behavior in the presence of a frequency modulated input. We have

$$m(t) = A_m \cos(2\pi f_m t)$$

with the corresponding FM wave given by

$$s(t) = A_c \sin[2\pi f_c t + \beta \sin(2\pi f_m t)]$$

where β is the modulation index.

a. Give the equation for $\phi_1(t)$.

b. Let us write

$$\phi_e(t) = \phi_{e0} \cos(2\pi f_m t + \psi).$$

Give the equations for ϕ_{e0} and ψ. In your expression for ϕ_{e0} use the variable $\Delta f = \beta f_m$.

c. Plot the phase error amplitude ϕ_{e0} normalized with respect to $\Delta f / K_0$ versus the dimenionless parameter f_m/K_0.

d. For the loop to track the frequency modulation closely we require the phase error $\phi_e(t)$ to remain small within the linear region which implies $\sin[\phi_e(t)] \approx \phi_e(t)$ which is true if $\phi_e(t) \leq 0.5$ radians. Explain why this requires $\Delta f \leq 0.5K_0$.

Problem 2. This is a continuation of Problem 1. The output signal $v(t)$ of the loop is related to the phase error $\phi_e(t)$ by

$$v(t) = \frac{K_0}{k_v}\phi_e(t).$$

a. Let us write

$$v(t) = A_0 \cos(2\pi f_m t + \psi).$$

Give the equations for A_0 and ψ.

b. At what value of f_m will the amplitude of the loop output $v(t)$ have fallen by 3 dB?

c. Show that the input frequency range over which the loop will hold lock is $\pm K_0$.

Problem 3. Consider an FM wave of carrier frequency f_c which is produced by a modulating wave $m(t)$. Assume that f_c is large enough to justify treating this FM wave as a narrow-band signal.

a. Find an approximate expression for its Hilbert transform.

b. For the special case of a sinusoidal wave $m(t) = A_m \cos(2\pi f_m t)$, find the exact expression for the Hilbert transform of the resulting FM wave. For this case, what is the error in the approximation used in part (a)?

Problem 4. Let $X_k,\ k = 1, 2, \ldots$ be independent Bernoulli distributed random variables such that

$$P(X_k = 1) = p, \quad P(X_k = 0) = 1 - p, \quad k = 1, 2, \ldots, \quad 0 < p < 1.$$

Let

$$Y_m = \prod_{k=1}^{m} X_k, \quad m \geq 1.$$

a. Compute $E[Y_m]$ for any $m \geq 1$ and also find $\lim_{m\to\infty} E[Y_m]$.

b. Compute $Var(Y_m)$ for any $m \geq 1$ and also find $\lim_{m\to\infty} Var(Y_m)$.

c. Compute $E[Y_m Y_n]$ for any $m, n \geq 1$ and also deduce the limiting behavior of $E[Y_m Y_n]$ as m and n tend to infinity.