EE 567

Homework 4 Solution

Problem 1. Over the interval $0 \le t \le 1$ a PM signal is given by $m(t)$

$$s_{PM}(t) = 10\cos(13,000\pi t).$$

It is known that the carrier frequency is 5000 Hz. If $k_p = 1000$ rad/volt determine $m(t)$ over the interval $0 \le t \le 1$.

Solution: An PM signal has phase $\theta(t)$ which is linear to the modulating signal $m(t)$, i.e.,

$$\theta(t) = 2\pi f_c t + k_p m(t)$$

Hence, with the given $s_{PM}(t) = A_c \cos(2\pi f_c t + k_p m(t)) = 10\cos(13,000\pi t)$, we have

$$m(t) = \frac{13,000\pi t - 2\pi f_c t}{k_p} = 3\pi t$$

where $0 \le t \le 1$.

Problem 2. Over the interval $0 \le t \le 1$ an FM signal is given by

$$s_{FM}(t) = 10\cos(13,000\pi t).$$

It is known that the carrier frequency is 5000 Hz. If $k_f = 1000$ Hz/volt determine $m(t)$ over the interval $0 \le t \le 1$.

Solution: An FM signal has instantaneous frequency linear to the modulating signal $m(t)$, i.e.

$$f_i(t) = f_c + k_f m(t)$$

The generalized angle of the FM signal $s_{FM}(t) = A_c \cos(\theta(t))$ is $\theta(t) = 2\pi t f_c t + 2\pi k_f \int_0^t m(s)ds$. Hence, we have

$$2\pi k_f \int 0tm(s)ds = 13,000\pi t - 2\pi f_c t = 3,000\pi t$$

$$\Rightarrow \int_0^t m(s)ds = \frac{3}{2}t$$

$$\Rightarrow m(t) = \frac{3}{2}$$

where $0 \leq t \leq 1$.

Note: You shouldn't write the generalized angle $\theta(t) = 2\pi t f_c t + k_f \int_0^t m(s)ds$ instead of $\theta(t) = 2\pi t f_c t + 2\pi k_f \int_0^t m(s)ds$, since the unit of frequency sensitivity k_f is Hz/volt.

Problem 3. An angle modulated signal is described by

$$s(t) = 20\cos\left(2\pi f_c t + 0.2\sin(2\pi f_1 t)\right)$$

where $f_c = 1$ MHz and $f_1 = 2$ kHz.

a. Find the power of the modulated signal $s(t)$.

b. Find the frequency deviation, Δf.

Solution:

a. Since the PM signal $s(t)$ has amplitude 20, the power of $s(t)$ is

$$P = \frac{20^2}{2} = 200$$

Note: The power of PM/FM signal does not change with different modulating signal $m(t)$.

b. In order to find the frequency deviation Δf, we need to compute the instantaneous frequency $f_i(t)$,

$$\theta(t) = 2\pi f_c t + 0.2\sin(2\pi f_1 t)$$

$$\Rightarrow f_i(t) = \frac{1}{2\pi}\frac{d\theta(t)}{dt} = f_c + 0.2 f_1 \cos(2\pi f_1 t)$$

$$\Rightarrow \Delta f = 400 \text{ Hz}$$

Problem 4. A carrier wave of frequency 25 MHz is frequency-modulated by a sine-wave of amplitude 5 volts and frequency 10 kHz. The frequency sensitivity of the modulator is 10 kHz per volt.

a. Determine the approximate bandwidth of the FM wave using Carson's rule.

b. Repeat part (a) assuming that the amplitude of the modulating wave is doubled.

c. Repeat part (a) assuming that the modulation frequency is doubled.

Solution:

Let the modulating signal be $m(t) = 5\cos(2\pi f_m t)$, where $f_m(t) = 10^4$ Hz and the instantaneous frequency $f_i(t) = f_c + k_f m(t)$, where $f_c = 2.5 \times 10^7$ Hz and $k_f = 10^4$ Hz/V.

$$\begin{aligned}\theta(t) &= 2\pi f_c t + 2\pi k_f \int_0^t m(s)ds \\ &= 2\pi f_c t + \frac{5k_f}{f_m}\sin(2\pi f_m t)\end{aligned}$$

Hence, the modulated signal is

$$s(t) = A_c \cos(2\pi f_c t + \frac{5k_f}{f_m}\sin(2\pi f_m t))$$

where the modulation index $\beta = \frac{5k_f}{f_m}$.

a. Carson's rule tells us that the effective bandwidth of the FM signal is approximately $B_{FM} = 2f_m(1+\beta)$. Thus,

$$B_{FM} = 2 \times 10^4 \times (1 + \frac{5 \times 10^4}{10^4}) = 1.2 \times 10^5 \text{ Hz.}$$

b.

$$B_{FM} = 2 \times 10^4 \times (1 + \frac{10 \times 10^4}{10^4}) = 2.2 \times 10^5 \text{ Hz.}$$

c.

$$B_{FM} = 4 \times 10^4 \times (1 + \frac{5 \times 10^4}{2 \times 10^4}) = 1.4 \times 10^5 \text{ Hz.}$$