# EE 567

## Homework 2 Solution

**Work all 5 problems**.

**Problem 1.**

a. Suppose $x(t)$ has the value 1 in the interval [0,2] and is zero elsewhere. Compute the energy in $x(t)$.

b. Suppose $y(t)$ has the value 1 in the interval [0,1] and has the value -1 in the interval (1,2] and is zero elsewhere. Compute the energy in $y(t)$.

**Solution:**

a. The energy of the signal $x(t)$ is:

$$E_x = \int_{-\infty}^{+\infty} x^2(t)dt = \int_0^2 1^2 dt = 2$$

b. The energy of the signal $y(t)$ is:

$$E_y = \int_{-\infty}^{+\infty} y^2(t)dt = \int_0^1 1^2 dt + \int_1^2 (-1)^2 dt = 2$$

**Problem 2.**

a. Suppose $x(t) = \sin(t)$ in the interval $[0, 2\pi]$ and is zero elsewhere. Compute the energy in $x(t)$.

b. Suppose $y(t) = A\sin(t)$ in the interval $[0, 2\pi]$ and is zero elsewhere. Compute the energy in $y(t)$.

**Solution:**

a. The energy of the signal $x(t)$ is:

$$E_x = \int_{-\infty}^{+\infty} x^2(t)dt = \int_0^{2\pi} \sin^2(t)dt = \int_0^{2\pi} \frac{1}{2}(1 - \cos 2t)dt = \pi$$

b. The energy of the signal $y(t)$ is:

$$E_y = \int_{-\infty}^{+\infty} y^2(t)dt = \int_0^{2\pi} A^2 \sin^2(t)dt = A^2 \int_0^{2\pi} \sin^2(t)dt = \pi A^2$$

**Problem 3.** Determine the power for each of the following signals:

a. $x(t) = 10\cos(100t + \pi/3)$.

b. $y(t) = (10 + 2\sin(3t))\cos(10t)$.

**Solution:**

Average the energy over a period of the signal to find the power for that signal.

a. The power of the signal $x(t)$ is:

$$\begin{aligned} P_x &= \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} 100\cos^2(100t + \pi/3)dt \\ &= \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} 50[1 + \cos(200t + 2\pi/3)]dt \\ &= 50 \end{aligned}$$

b. The power of the signal $y(t)$ is:

$$\begin{aligned} P_y &= \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} [10\cos 10t + 2\sin 3t \cos 10t]^2 dt \\ &= \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} [10\cos 10t + \sin 13t - \sin 7t]^2 dt \\ &= 100/2 + 1/2 + 1/2 = 51 \end{aligned}$$

**Problem 4.** Pulse Coded Modulation (PCM) is to be used to encode a signal. The signal ranges between the values -2 and +2. There are 4 bits or 16 levels (hence 16 code numbers) available. The levels assigned have symmetry like we demonstrated in class. The first three sample values obtained (before quantization) are 0.6, 1.4, and -1.2, respectively.

a. Find the quantized values for the three sample values.

b. Find the corresponding PCM sequences for the quantized values.

**Solution:**

| Bin Interval | Quantized Value | PCM Codeword |
|---|---|---|
| [-2, -1.75) | -1.875 | 0000 |
| [-1.75, -1.5) | -1.625 | 0001 |
| [-1.5, -1.25) | -1.375 | 0010 |
| [-1.25, -1) | -1.125 | 0011 |
| [-1, -0.75) | -0.875 | 0100 |
| [-0.75, -0.5) | -0.625 | 0101 |
| [-0.5, -0.25) | -0.375 | 0110 |
| [-0.25, 0) | -0.125 | 0111 |
| [0, 0.25) | 0.125 | 1000 |
| [0.25, 0.5) | 0.375 | 1001 |
| [0.5, 0.75) | 0.625 | 1010 |
| [0.75, 1) | 0.875 | 1011 |
| [1, 1.25) | 1.125 | 1100 |
| [1.25, 1.5) | 1.375 | 1101 |
| [1.5, 1.75) | 1.625 | 1110 |
| [1.75, 2] | 1.875 | 1111 |

Table 1: PCM Table

a. Given the sample values $\{0.6, 1.4, -1.2\}$, we can find the quantized values with the above table as $\{0.625, 1.375, -1.125\}$.

b. The corresponding PCM sequence is $\{1010, 1101, 0011\}$.

**Problem 5.** Let $s(t) = 10\cos(2\pi f t + \pi/4)$ where $f = 40$ Hz. Let us sample $s(t)$ at the sampling rate of $f_s = 160$ Hz to obtain the discrete time signal $s(nT_s) = 10\cos(2\pi f n T_s + \pi/4)$ where $T_s = 1/f_s$, for $n = 0, 1, 2, \ldots, 40$. Using the PCM example in class as a guide compute the quantized PAM signal and the corresponding PCM codeword assuming you have 8 bits or 256 levels to represent the quantized signal.

Note: In this problem you are to use Matlab or some other programming tool/language. Remember to hand in your software code.

**Solution:**

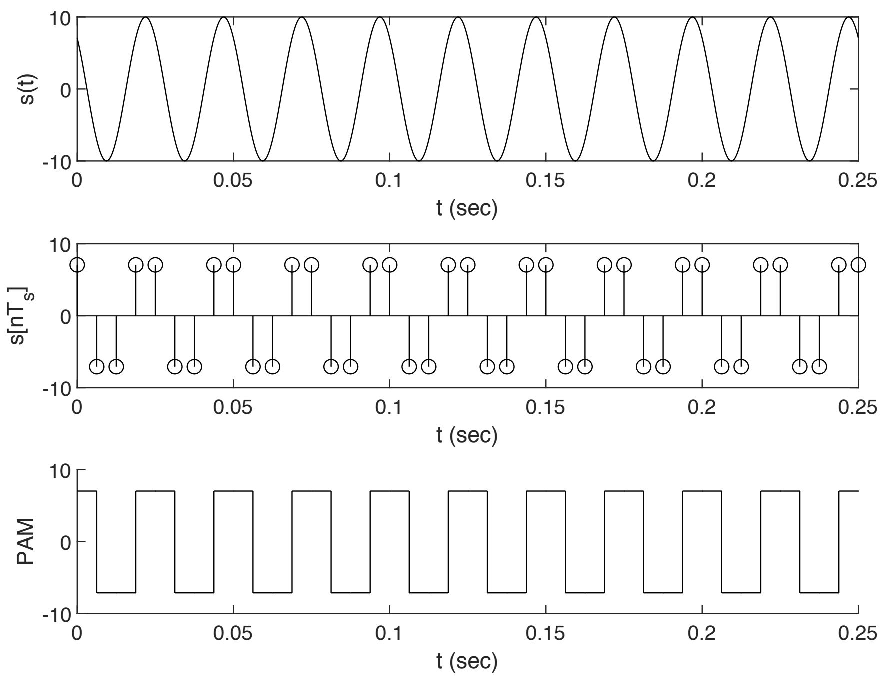


Figure 1: Original signal $s(t)$, its discrete-time sample $s[nT_s]$ with $f_s = 160$ Hz and the corresponding PAM signal.

Observe that after sampling $s(t)$ with sampling rate $f_s = 160$ Hz, we get a discrete signal $s[nT_s]$ with only two values, 7.0711 and -7.0711 which correspond to code number 218 (quantized value 7.0703) and code number 37 (quantized value -7.0703), respectively.

Hence, the PCM sequence is $\{218, 37, 37, 218, \cdots\}$ or with binary representation $\{11011010, 00100101, 00100101, 11011010, \cdots\}$.

**Appendix: MATLAB code**

**Problem 5:**

```
clear; clc; close all;
f=40; fs=160;
Ts=1/fs;
t=linspace(0,1/f*10,1001)';
st=10*cos(2*pi*f*t+pi/4);
n=[0:40]';
sn=10*cos(2*pi*f*n*Ts+pi/4);
figure, hold on;
subplot(3,1,1); plot(t,st,'k'); xlabel('t (sec)'); ylabel('s(t)');
subplot(3,1,2); stem(n*Ts,sn,'k'); xlabel('t (sec)'); ylabel('s[n]');
L=256;
subplot(3,1,3); hold on;
kn=floor((sn+10)/(20/L));
sn=-10+(20/L)*kn;
for i=1:length(sn)-1
   plot(Ts*[n(i),n(i+1)],[sn(i),sn(i)],'k');
   plot(Ts*[n(i+1),n(i+1)],[sn(i),sn(i+1)],'k');
end
xlabel('t (sec)'); ylabel('PAM');
```