EE 567 Final Exam Part 2 Solution

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Problem	Points	Score
5	20	
6	18	
7	12	
Part 2	50	
Part 1	50	
Parts 1+2	100	

Instructions and Information:

1) Print your name and indicate on-campus or DEN student at the top of the page.

2) Make sure this part of the exam has 3 problems.

3) This is a closed book exam. You may use two 8.5x11 inch sheet of notes (front and back). You may use a self-contained calculator but not a computer. Cell phones or any device with internet capability is not permitted. **You have 55 minutes to take this part of the exam**.

4) The points for each problem is shown above.

Problem 5.

Suppose we receive a BPSK signal of the form

$$r(t) = A\cos(2\pi f_c t + \theta) + n(t), \quad 0 \le t \le T$$

where $n(t)$ is an AWGN process and $A = \pm 1$ equally likely. The signal is demodulated in an optimal manner and then an optimal threshold is used in making a decision. For ideal demodulation the received signal $r(t)$ is mixed with a cosine wave with the same frequency and phase of the transmitted signal of interest and then low pass filtered by integrating from 0 to T. This is coherent detection with a matched filter. With this approach we derived in class the probability of bit error as

$$P_b = Q\left(\sqrt{\frac{2E_b}{N_0}}\right).$$

a. Suppose in the mixing signal the receiver does not use the exact correct frequency but instead uses $\cos(2\pi(f_c+\Delta f)t+\theta)$. What is the resulting P_b in this case? You may assume high frequency terms are completely removed by the low pass filter. One way to approach this is to compute the SNR degradation due to Δf resulting after mixing and low pass filtering and then adjust the ideal P_b formula based on this.

Solution: Ideally, after mixing and low pass filtering the signal component is

$$\textit{signal component} = \frac{AT}{2}.$$

With the frequency offset we get

$$\begin{aligned} \textit{signal component} &= \frac{A}{2}\int_0^T \cos(2\pi\Delta f t)dt \\ &= \frac{A\sin(2\pi\Delta f T)}{4\pi\Delta f}. \end{aligned}$$

Hence, the SNR degradation is

$$\textit{SNR degradation} = \frac{\sin^2(2\pi\Delta f T)/(2\pi\Delta f)^2}{T^2} = \frac{\sin^2(2\pi\Delta f T)}{(2\pi\Delta f T)^2}.$$

Thus,

$$P_b = Q\left(\sqrt{\frac{2E_b\frac{\sin^2(2\pi\Delta fT)}{(2\pi\Delta fT)^2}}{N_0}}\right) = Q\left(\sqrt{\frac{2E_b}{N_0}}\cdot\frac{\sin(2\pi\Delta fT)}{2\pi\Delta fT}\right).$$

b. Suppose now that $\Delta f = 0$ but we have a pulse jammer present that generates zero-mean wide sense stationary broadband noise when present. The fraction of the time the jammer is present is ρ with $0 \leq \rho \leq 1$. If the jammer was on all the time ($\rho = 1$) its power spectral density would be J_0. Give an expression for average P_b in this case with arbitrary ρ. For simplicity, you may assume when the jammer is present it is on for an integer number of BPSK symbol times and starts and stops on symbol boundaries.

Solution:

$$P_b = (1-\rho)\cdot Q\left(\sqrt{\frac{2E_b}{N_0}}\right) + \rho\cdot Q\left(\sqrt{\frac{2E_b}{N_0 + J_0/\rho}}\right).$$

c. Simplify your answer in (b) assuming that $J_0 \gg N_0$. This answer will be an approximation.

Solution:

$$P_b \approx \rho\cdot Q\left(\sqrt{\frac{2\rho E_b}{J_0}}\right).$$

Problem 6. A binary communication system is used where at the receiver the test statistic is $z(T) = A + n_0$, where $A = \pm 1$ if a signal is present and $A = 0$ if a signal is not present. The noise n_0 is distributed as

$$p(n_0) = \begin{cases} \frac{n_0}{4} + \frac{1}{2}, & -2 \leq n_0 \leq 0, \\ \frac{1}{2} - \frac{n_0}{4}, & 0 < n_0 \leq 2, \\ 0 \text{ elsewhere.} \end{cases}$$

However, the receiver is just interested in detecting whether or not the signal is actually present. If the signal is not present the noise n_0 is still present and if the signal is present the noise is present also. So, the receiver designer decides to take the absolute value of the test statistic and compare this to a threshold. If $|Z(T)| > T_0$, for some T_0, then a signal is declared present, otherwise, it is not.

a. Find T_0 such that the probability of false alarm is 10^{-2}.

   Solution: The noise density is a triangle with vertices (-2,0), (0,1/2), (2,0). The absolute value of the noise then is a triangle with vertices (0,0), (0,1), (2,0). The equation of the line connecting the vertex (0,1) with (2,0) is $y = -0.5x + 1$. So the value of the absolute noise at T_0 is $1 - \frac{T_0}{2}$. So we solve

$$\frac{1}{2}\left(1 - \frac{T_0}{2}\right)(2 - T_0) = 10^{-2}$$

   to get $T_0 = 2(1 - \sqrt{10^{-2}}) = 1.8$.

b. If a signal is present find the probability that you would detect it with this logic if the value of $T_0 = 1.75$ (which is not the answer to part (a)).

   Solution: If a signal is present we may assume WLOG that $A = 1$. The signal plse noise density then is a triangle with vertices (-1,0), (1,1/2), (3,0). When we take the absolute value of the signal plus noise the part of the signal plus noise density from -1 to 0 on the x-axis folds over to the positive x-axis from 0 to 1. But we do not have to consider this since we are interested in the region from 1.75 to 3 on the x-axis. The equation of the line connecting the vertex (1,1/2) to (3,0) is $y = -0.25x + 0.75$ which evaluates to $y = 0.3125$ when $x = 1.75$. So we compute

$$P_d = \frac{1}{2} \cdot 0.3125 \cdot (3 - 1.75) = 0.1953.$$

Problem 7. Short answers (no detailed derivations needed).

a. Suppose in an error correction scheme we have 4 information bits mapped to 7 encoded bits. What is the resulting code rate of this code?

Solution: rate $= \frac{4}{7}$.

b. What advantage do we gain in transmitting communication signals at high frequencies instead of low frequencies?

Solution: We can utilize a smaller antenna for the same performance.

c. What is the smallest theoretical value that a noise figure for a device can achieve (that is, what is the greatest lower bound on the noise figure)?

Solution: The NF cannot be smaller than 1.

d. In cascading filters in a receiver do you want to place most of the gain in the first filter or the last filter in the cascaded chain (or does it matter)?

Solution: First filter.

e. Specify one purpose of a phase-lock loop.

Solution: Can use it for FM demodulation.

f. Does M of N filtering have slightly better or slightly worse performance than an integration filter or is it the same performance?

Solution: It is slightly worse performing.