

# EE 567

## Homework 7

Due Monday, October 16, 2017

**Work all 3 problems.**

**Problem 1.** In class we had that the gain for a parabolic antenna is

$$g = \left(\frac{\pi}{3}\right)^2 \left(\frac{df_c}{10^8}\right)^2$$

and the half-power beamwidth is

$$\phi_b = 3.06 \left(\frac{10^8}{df_c}\right) \text{ radians.}$$

The actual gain of the antenna will be reduced if there is any pointing error  $\phi_e$  and the resulting equation is

$$g(\phi_e) = g \left[ \frac{2J_1(\pi d\phi_e/\lambda)}{(\pi d\phi_e/\lambda)} \right]$$

where  $\lambda$  is the wavelength. Using the Bessel function approximation for small argument

$$J_1(x) \approx \frac{x}{2} \left(1 - \frac{x^2}{8}\right) \approx \frac{x}{2} e^{-x^2/8}$$

we obtain

$$g(\phi_e) \approx g e^{-2.6(\phi_e/\phi_b)^2}.$$

Plot the antenna gain (in dB) versus  $d/\lambda$  for  $\phi_e = 0, 0.01, 0.05, 0.1, 0.15$  degrees. You should plot each of these on the same graph. Vary  $d/\lambda$  from 10 to 3000. You should be able to deduce from your results that pointing errors prevent the use of very narrow beams.

**Problem 2.** A signal of the form

$$s_1(t) = A \cos(2\pi f_c t + \theta)$$

is transmitted to a receiver. The signal waveform is  $T$  seconds long. A multipath version of the signal,  $s_2(t)$ , delayed by  $\tau$  seconds also arrives at the receiver (you can think of this multipath signal as being just like  $s_1(t)$  except it is delayed and the amplitude is possibly scaled). The cross-correlation of the two real signals is defined by

$$\gamma_{12} = \frac{1}{T} \int_0^T s_1(t) s_2(t) dt.$$

Determine the smallest value of  $\tau$  that can be tolerated to ensure that the cross-correlation of the direct path and multipath signal is less than 5% of the direct signal power. Assume that  $f_c T \gg 1$  and the multipath signal has an amplitude that is 70% that of the direct signal when received.

**Problem 3.** Use Matlab. Suppose we receive the analog signal

$$r_a(t) = A \cos(2\pi 200t + \theta)$$

and sample it at 2000 Hz to get the digital signal

$$r(n) = A \cos(0.2\pi n + \theta).$$

Suppose now we quantize the digital signal to get

$$r_q(n) = \text{Round}[A \cos(0.20\pi n + \theta)]$$

where ‘Round’ means the samples are rounded to the nearest integer. The amplitude  $A$  is a constant but we do not know its value. Furthermore, we do not know that the phase is  $\theta = \pi/4$ . We can follow the steps below to estimate the value of  $A$ . [For purposes of calculation let  $A$  actually have the value 10.]

- S1.** Multiply  $r_q(n)$  by  $x_1(n)$  and  $x_2(n)$ , where  $x_1(n) = \cos(0.2\pi n)$  and  $x_2(n) = \sin(0.2\pi n)$ . Call the results  $y_1(n)$  and  $y_2(n)$ , respectively.
- S2.** Simply add up the values of  $y_1(n)$  and  $y_2(n)$  for  $n = 0, 1, 2, \dots, N - 1$  (some  $N$ ) and take the average of each (divide by  $N$ ) and then multiply the averages by 2. Call the results  $z_1$  and  $z_2$ , respectively.

- S3.** Compute  $\sqrt{z_1^2 + z_2^2}$ . This is the estimate of  $A$ .
- Follow the 3 steps above and estimate  $A$  using  $N = 5$ .
  - Repeat (a) for  $N = 10$ .
  - What values of  $N$  makes your estimate of  $A$  exact if the input samples were not quantized.
  - Based on your answers to parts (a) and (b) what would your estimate for  $A$  be if  $N$  is 1000. Explain why the estimate is not becoming exact even for very large  $N$ .
  - If we change the sampling frequency to  $2000 \times \pi/3$  Hz will our estimate for  $A$  become exact for large  $N$ ? If so, explain why and also show that the estimate becomes exact using Matlab.