

# EE 567

## Homework 6

Due Monday, October 9, 2017

**Work all 4 problems.**

**Problem 1.** Consider an ideal bandpass filter

$$H_{BP}(f) = \begin{cases} e^{-j2\pi(f-f_c)t_0}, & f_c - B \leq f \leq f_c + B \\ e^{-j2\pi(f+f_c)t_0}, & -f_c - B \leq f \leq -f_c + B \\ 0, & \text{elsewhere.} \end{cases}$$

Let  $s(t) = Ar_T(t) \cos(2\pi f_c t)$  where

$$r_T(t) = \begin{cases} 1, & -T/2 \leq t \leq T/2 \\ 0, & \text{elsewhere.} \end{cases}$$

Determine the response of the bandpass system to the input  $s(t)$ . Your answer may utilize the sine integral where the sine integral is defined by

$$\text{Si}(u) = \int_0^u \frac{\sin \lambda}{\lambda}.$$

You may assume that  $f_c T \gg 1$  so that  $s(t)$  may be considered narrowband.

**Problem 2.** Let  $X_k$ ,  $k = 1, 2, \dots, n$  be a sequence of independent and identically distributed random variables each with mean  $\mu$  and variance  $\sigma^2$ . Let

$$S^2 = \frac{1}{n-1} \sum_{k=1}^n (X_k - \bar{X})^2.$$

Show that  $E[S^2] = \sigma^2$  (do not assume that  $X_k$  is Gaussian).

*Hint:*  $X_k - \bar{X} = X_k - \mu + \mu - \bar{X}$ .

**Problem 3.** Suppose  $X_1, X_2, \dots$  are each independent and normally distributed with mean zero and variance one (standard normal). Define  $Y_1 = X_1$  and for  $n = 2, 3, 4, \dots$  let

$$Y_n = \alpha X_n + (1 - \alpha)Y_{n-1}, \quad 0 < \alpha < 1.$$

- Compute  $E[Y_n Y_m]$  as a closed form function of  $n, m$  and  $\alpha$  for  $n \geq m$ .
- Evaluate your expression for  $E[Y_n Y_m]$  using  $n = 10, m = 5$  and  $\alpha = 0.2$ .

**Problem 4.** Consider a real Gaussian random sequence  $x(n)$ ,  $n$  an integer, with

$$E[x(n)] = 0, \quad E[x(n)^2] = 1, \quad E[x(n)x(m)] = \rho^{|n-m|}$$

where  $0 < \rho < 1$ . Let

$$y(n) = 2x(n) + 2.$$

- Is  $x(n)$  wide sense stationary?
- Find the covariance of  $y(n)$  and state whether or not it is wide sense stationary.

Note: WSS has the same meaning here using the index  $n$  as in the lecture notes where we used  $t$  but now the “time” is discrete.