

# EE 567

## Homework 4 Solution

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**Problem 1.** Lathi and Ding 5.1-4 (modified). Over the interval  $0 \leq t \leq 1$  a PM signal is given by

$$s_{PM}(t) = 10 \cos(13,000\pi t).$$

It is known that the carrier frequency is 5000 Hz. If  $k_p = 1000$  rad/volt determine  $m(t)$  over the interval  $0 \leq t \leq 1$ .

**Solution:**

An PM signal has phase  $\theta(t)$  which is linear to the modulating signal  $m(t)$ , i.e.

$$\theta(t) = 2\pi f_c t + k_p m(t).$$

Hence, with the given  $s_{PM} = A_c \cos(2\pi f_c t + k_p m(t)) = 10 \cos(13000\pi t)$ , we have

$$\begin{aligned} m(t) &= \frac{13000\pi t - 2\pi f_c t}{k_p} \\ &= 3\pi t, \text{ where } t \in [0, 1]. \end{aligned}$$

**Problem 2.** Lathi and Ding 5.1-4 (modified). Over the interval  $0 \leq t \leq 1$  an FM signal is given by

$$s_{FM}(t) = 10 \cos(13,000\pi t).$$

It is known that the carrier frequency is 5000 Hz. If  $k_f = 1000$  Hz/volt determine  $m(t)$  over the interval  $0 \leq t \leq 1$ .

**Solution:**

An FM signal has instantaneous frequency linear to the modulating signal  $m(t)$ , i.e.

$$f_i(t) = f_c + k_f m(t).$$

The generalized angle of the FM signal  $s_{FM}(t) = A_c \cos(\theta(t))$  is  $\theta(t) =$

$2\pi f_c t + 2\pi k_f \int_0^t m(s) ds$ . Hence, we have

$$\begin{aligned} 2\pi k_f \int_0^t m(s) ds &= 13000\pi t - 2\pi f_c t = 3000\pi t \\ \Rightarrow \int_0^t m(s) ds &= \frac{3}{2}t \\ \Rightarrow m(t) &= \frac{3}{2}, \text{ where } t \in [0, 1] \end{aligned}$$

**Note:** You shouldn't write the generalized angle  $\theta(t) = 2\pi f_c t + k_f \int_0^t m(s) ds$  instead of  $\theta(t) = 2\pi f_c t + 2\pi k_f \int_0^t m(s) ds$ , since the unit of frequency sensitivity  $k_f$  is Hz/volt.

**Problem 3.** An angle modulated signal is described by

$$s(t) = 20 \cos(2\pi f_c t + 0.2 \sin(2\pi f_1 t))$$

where  $f_c = 1$  MHz and  $f_1 = 2$  kHz.

- Find the power of the modulated signal  $s(t)$ .
- Find the frequency deviation,  $\Delta f$ .

**Solution:**

- Since the PM signal  $s(t)$  has amplitude 20, the power of  $s(t)$  is

$$P = \frac{20^2}{2} = 200.$$

**Note:** The power of PM/FM signal does not change with different modulating signal  $m(t)$ .

- In order to find the frequency deviation  $\Delta f$ , we need to compute the instantaneous frequency  $f_i(t)$ ,

$$\begin{aligned} \theta(t) &= 2\pi f_c t + 0.2 \sin(2\pi f_1 t) \\ \Rightarrow f_i(t) &= \frac{1}{2\pi} \frac{d\theta(t)}{dt} = f_c + 0.2 f_1 \cos(2\pi f_1 t) \\ \Rightarrow \Delta f &= 400 \text{ Hz.} \end{aligned}$$

**Problem 4.** A carrier wave of frequency 25 MHz is frequency-modulated by a sine-wave of amplitude 5 volts and frequency 10 kHz. The frequency sensitivity of the modulator is 10 kHz per volt.

- Determine the approximate bandwidth of the FM wave using Carson's rule.
- Repeat part (a) assuming that the amplitude of the modulating wave is doubled.
- Repeat part (a) assuming that the modulation frequency is doubled.

**Solution:**

Let the modulating signal be  $m(t) = 5 \cos(2\pi f_m t)$ , where  $f_m(t) = 10^4$  Hz and the instantaneous frequency  $f_i(t) = f_c + k_f m(t)$ , where  $f_c = 2.5 \times 10^7$  Hz and  $k_f = 10^4$  Hz/V.

$$\begin{aligned}\theta(t) &= 2\pi f_c t + 2\pi k_f \int_0^t m(s) ds \\ &= 2\pi f_c t + \frac{5k_f}{f_m} \sin(2\pi f_m t).\end{aligned}$$

Hence, the modulated signal is

$$s(t) = A_c \cos\left(2\pi f_c t + \frac{5k_f}{f_m} \sin(2\pi f_m t)\right),$$

where the modulation index  $\beta = \frac{5k_f}{f_m}$ .

- Carson's rule tells us that the effective bandwidth of the FM signal is approximately  $B_{FM} = 2f_m(1 + \beta)$ . Thus,

$$B_{FM} = 2 \times 10^4 \times \left(1 + \frac{5 \times 10^4}{10^4}\right) = 1.2 \times 10^5 \text{ Hz.}$$

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$$B_{FM} = 2 \times 10^4 \times \left(1 + \frac{10 \times 10^4}{10^4}\right) = 2.2 \times 10^5 \text{ Hz.}$$

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$$B_{FM} = 4 \times 10^4 \times \left(1 + \frac{5 \times 10^4}{2 \times 10^4}\right) = 1.4 \times 10^5 \text{ Hz.}$$