

EE 567

Homework 3 Solution

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Problem 1. You are given the baseband signal $m(t) = \cos(2000\pi t)$.

- Sketch the spectrum of $m(t)$.
- Sketch the spectrum of the DSB-SC signal $m(t) \cos(20,000\pi t)$.

Solution:

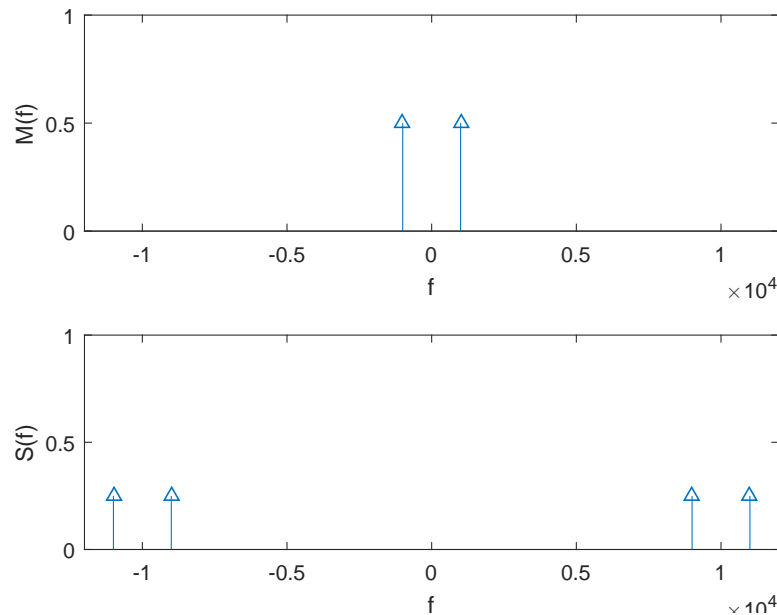


Figure 1: Spectrum of the message signal $m(t)$ and the modulated signal $s(t) = m(t) \cos(20000\pi t)$

Problem 2. Lathi and Ding 4.2-3 (modified). You are asked to design a DSB-SC to generate a modulated signal $s(t) = km(t) \cos(2\pi f_c t)$ where $m(t)$ is a signal band-limited to B Hz. One could simply multiply $km(t)$ by $\cos(2\pi f_c t)$ to accomplish this. However, suppose that we do not have available an oscillator to generate the signal $\cos(2\pi f_c t)$ but we do have available

an oscillator that can generate the signal $\cos^3(2\pi f_c t)$. We also have a band-pass filter that we can tune to any frequency and bandwidth. Can we still generate $s(t)$ with this equipment? If so, explain how.

Solution: Using the trigonometric identity,

$$\cos(3\theta) = 4 \cos^3(\theta) - 3 \cos(\theta),$$

we can generate a carrier signal $\cos(2\pi f_c t)$ with the availability of $\cos^3(2\pi f_c t)$ and a lowpass(or bandpass) filter since

$$\cos^3(2\pi f_c t) = \frac{1}{4} \cos(2\pi \cdot 3f_c t) + \frac{3}{4} \cos(2\pi f_c t).$$

Problem 3. In a DSB-SC amplitude modulation system the message signal is $m(t) = e^{-t}u(t - 2)$ and the carrier signal is $\cos(1000\pi t)$.

- Find the Fourier transform of the message signal and plot its magnitude.
- Find the Fourier transform of the modulated signal and plot its magnitude.

Solution: Calculate the Fourier transform of $m(t)$ directly as

$$\begin{aligned} M(f) &= \int_{-\infty}^{\infty} m(t)e^{-j2\pi ft} dt \\ &= \int_2^{\infty} e^{(-1-j2\pi f)t} dt \\ &= \frac{1}{-1-j2\pi f} e^{(-1-j2\pi f)t} \Big|_{t=2}^{\infty} \\ &= \frac{e^{-2-j4\pi f}}{1+j2\pi f}. \end{aligned}$$

- The Fourier transform of the message signal $m(t)$ is

$$M(f) = \frac{e^{-2-j4\pi f}}{1+j2\pi f},$$

and its magnitude plot in frequency domain

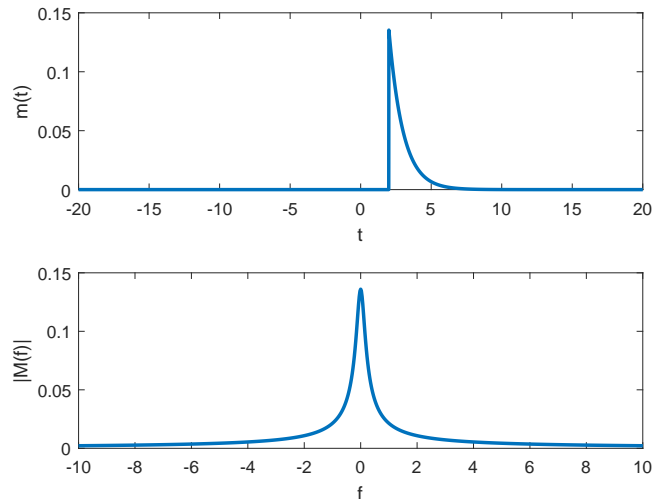


Figure 2: Magnitude of $M(f)$

- b. The Fourier transform of the modulated signal $s(t) = m(t) \cos(2\pi \cdot 500t)$ is

$$S(f) = \frac{1}{2} (M(f + 500) + M(f - 500)),$$

and its magnitude plot in frequency domain

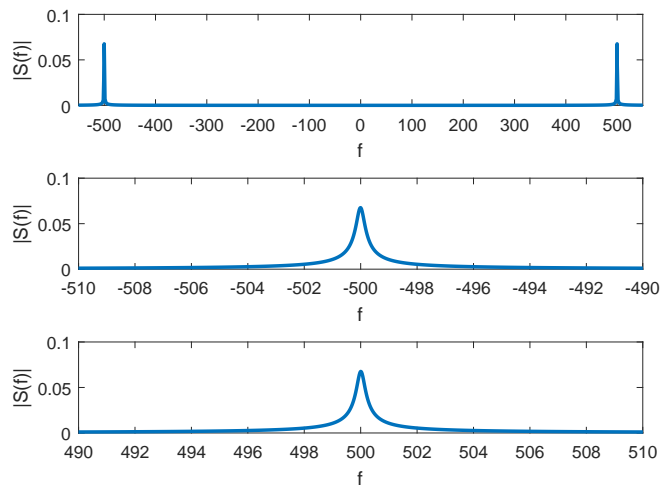


Figure 3: Magnitude of $S(f)$

Problem 4. In this problem you are to use Matlab. Suppose you transmit to someone the message signal

$$m(t) = \begin{cases} 1000t, & 0 \leq t \leq 1/2 \text{ msec}, \\ 1 - 1000t, & 1/2 < t \leq 1 \text{ msec}. \end{cases}$$

The actual transmission is accomplished by using DSB-SC via

$$s(t) = m(t) \cos(2\pi f_c t)$$

where, $f_c = 1$ MHz. This is a finite duration signal so it has an infinite nonzero frequency content. At the receiver a bandpass filter is applied to $s(t)$ so necessarily $m(t)$ cannot be recovered perfectly in an actual system. Suppose an ideal bandpass filter $H_B(f)$ is used with height unity centered at $\pm f_c$ that extends from $f_c - B$ to $f_c + B$ and $-f_c - B$ to $-f_c + B$.

Let us define a distortion measure as

$$D_B = 10 \cdot \log_{10} \frac{\int_{-\infty}^{\infty} |S(f)H_B(f)|^2 df}{\int_{-\infty}^{\infty} |S(f)|^2 df} \quad (\text{dB})$$

so that no distortion ($B = \infty$) corresponds to $D_B = 0$ dB.

- Plot D_B for $B = 10$ KHz down to $B = 100$ Hz.
- Determine the smallest value of B such that $D_B > -1$ dB.

Solution:

Notice that the amplified and time-shifted message signal $m'(t) = 2m(t + 0.0005)$ is equivalent to a triangular pulse $\text{trig}(2000t)$, where

$$\text{trig}(t) = \begin{cases} 1 + t, & \text{if } t \in [-1, 0] \\ 1 - t, & \text{if } t \in (0, 1] \\ 0, & \text{otherwise.} \end{cases}$$

We have the Fourier transform pair $\text{trig}(t) \leftrightarrow \text{sinc}^2(f)$ and thus $m'(t) = \text{trig}(2000t) \leftrightarrow \frac{1}{2000} \text{sinc}^2(\frac{f}{2000})$. Using the relationship mentioned above, we have

$$|M(f)|^2 = \frac{|M'(f)|^2}{4} = \frac{\text{sinc}^4(f/2000)}{4 \cdot 4 \cdot 10^6}.$$

With the assumption that an ideal bandpass filter $H_B(f)$ applied on the modulated signal $s(t)$, the distortion measure can be simplified further as

$$\begin{aligned} D &= 10 \log \left(\frac{\int_{-\infty}^{\infty} |S(f)H_B(f)|^2 df}{\int_{-\infty}^{\infty} |S(f)|^2 df} \right) \\ &= 10 \log \left(\frac{\int_{-B}^B |M(f)|^2 df}{\int_{-\infty}^{\infty} |M(f)|^2 df} \right) \\ &= 10 \log \left(12000 \int_{-B}^B |M(f)|^2 df \right), \end{aligned}$$

since the Parseval's theorem gives $\int_{-\infty}^{\infty} |M(f)|^2 df = \int_{-\infty}^{\infty} |m(t)|^2 dt = \frac{1}{12000}$.

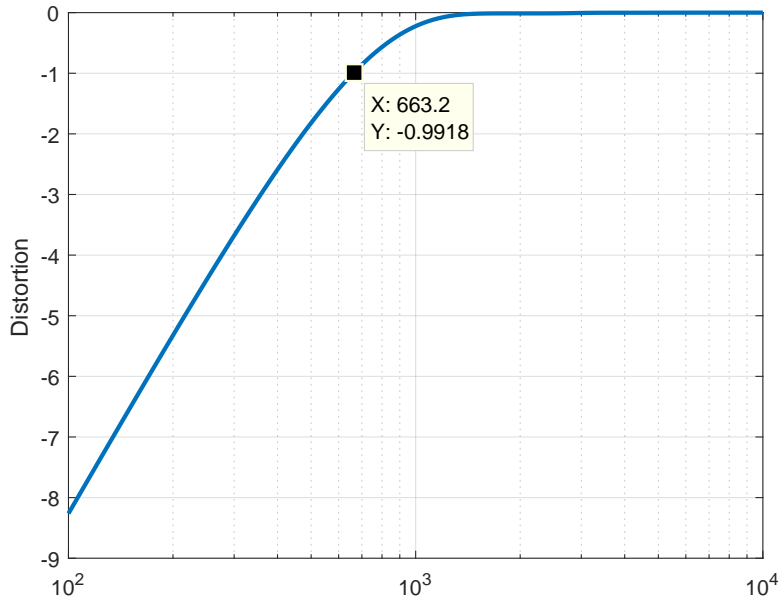


Figure 4: Distortion measure versus bandwidth of bandpass filter $H_B(f)$

The smallest bandwidth B that yields $D_B > -1$ dB is around 663Hz.