EE 567

Homework 2 Solution

Instructor: Christopher Wayne Walker, TA: James Huang

Problem 1. Lathi and Ding 2.1-2.

a. Find E_x and E_y, the energies of the signals $x(t)$ and $y(t)$ shown in Fig. P2.1-2a (see page 3). Sketch the signals $x(t) + y(t)$ and $x(t) - y(t)$. Show that the energies of either of these two signals is equal to $E_x + E_y$. Repeat the procedure for the signal pair in Fig. P2.1-2b (see page 3).

b. Now repeat the procedure for the signal pair in Fig. P2.1-2c (see page 3). Are the energies of the signals $x(t) + y(t)$ and $x(t) - y(t)$ identical in this case?

Solution:

a. The energies of signals $x(t)$ and $y(t)$ are $E_x = 2$ and $E_y = 2$. The energy of the sum of $x(t)$ and $y(t)$ is $E_{x+y} = 4$ and the energy of the difference of $x(t)$ and $y(t)$ is $E_{x-y} = 4$. In this case,

$$E_{x+y} = E_{x-y} = E_x + E_y.$$

For the signals in Fig. P2.1-2b, we observe that the energies of signals $x(t)$ and $y(t)$ are $E_x = 2\pi$ and $E_y = 2\pi$. The energy of the sum of $x(t)$ and $y(t)$ is $E_{x+y} = 4\pi$ and the energy of the difference of $x(t)$ and $y(t)$ is $E_{x-y} = 4\pi$. Hence,

$$E_{x+y} = E_{x-y} = E_x + E_y.$$

b. For the signals in Fig. P2.1-2c, while $E_x = \pi$ and $E_y = \pi$, we find that $E_{x+y} = \pi$ and $E_{x-y} = 3\pi$. Therefore, the sum of individual energies E_x and E_y is neither the same as the energy of the sum of signals E_{x+y} nor the same as the energy of the difference of signals E_{x-y}.

Problem 2. Lathi and Ding 2.1-3. Redo Example 2.2a to find the power of a sinusoid $C\cos(\omega_0 t + \theta)$ by averaging the signal energy over one period $2\pi/\omega_0$ (rather than averaging over the infinitely large interval).

Example 2.2a: Determine the power and rms value of $g(t) = C\cos(\omega_0 t + \theta)$.

Solution:

Find the power of $g(t) = C\cos(\omega_0 t+\theta)$ by averaging the energy over a period $T = 2\pi/\omega_0$.

$$\begin{aligned}
P_g &= \frac{1}{T}\int_0^T g^2(t)dt \\
&= \frac{\omega_0}{2\pi}\int_0^{\frac{2\pi}{\omega_0}} C^2\cos(\omega_0 t + \theta)dt \\
&= \frac{\omega_0}{2\pi}\int_0^{\frac{2\pi}{\omega_0}} \frac{C^2}{2}\left[1 + \cos\left(2\omega_0 t + 2\theta\right)\right]dt \\
&= \frac{C^2}{2}.
\end{aligned}$$

The second part of the integral is zero since it is integrating a sinusoid signal over integer periods.

Problem 3. Lathi and Ding 2.1-6. Find the energies of the signals shown in Fig. 2.1-6. Comment on the effect on energy of sign change, time shift, or doubling of the signal. What is the effect on the energy if the signal is multiplied by k?

Solution:
The energy of the signal $g(t)$ shown in Fig. 2.1-6 is

$$E_g = \int_0^{2\pi} \sin^2(t)dt = \int_0^{2\pi} \frac{1}{2}\left(1 - \cos 2t\right)dt = \pi.$$

The energy remains unchanged if sign-changed and time-shifted. However, doubling the amplitude multiplies the energy by a factor of 4. If the signal $g(t)$ is multiplied by k, then the energy is multiplied by k^2.

Problem 4. Lathi and Ding 2.1-8 (a), (b), (c). Determine the power and the rms value for each of the following signals:

a. $10\cos\left(100t + \frac{\pi}{3}\right)$

b. $10\cos\left(100t + \frac{\pi}{3}\right) + 16\sin\left(150t + \frac{\pi}{5}\right)$

c. $(10 + 2\sin 3t)\cos 10t$

Solution:

a.

$$\begin{aligned} P &= \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} 100\cos^2\left(100t + \frac{\pi}{3}\right) dt \\ &= \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} 50\left[1 + \cos\left(200t + \frac{2\pi}{3}\right)\right] dt \\ &= 50. \end{aligned}$$

Then the rms value is $\sqrt{50}$.

b.

$$\begin{aligned} P = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} & 100\cos^2\left(100t + \frac{\pi}{3}\right) + 320\cos\left(100t + \frac{\pi}{3}\right)\sin\left(150t + \frac{\pi}{5}\right) \\ & + 256\sin^2\left(150t + \frac{\pi}{5}\right) dt \\ = {} & 50 + 0 + 128 \\ = {} & 178. \end{aligned}$$

The second term in the integral is 0 due to the trigonometric product identity:

$$\cos\alpha\sin\beta = \frac{1}{2}\left(\sin(\alpha+\beta) - \sin(\alpha-\beta)\right).$$

Then the rms value is $\sqrt{178}$.

c.

$$\begin{aligned} P &= \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} \left[10\cos 10t + 2\sin 3t\cos 10t\right]^2 dt \\ &= \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} \left[10\cos 10t + \sin 13t - \sin 7t\right]^2 dt \\ &= \frac{100}{2} + \frac{1}{2} + \frac{1}{2} \\ &= 51. \end{aligned}$$

Then the rms value is $\sqrt{51}$.

Problem 5. Pulse Coded Modulation (PCM) is to be used to encode a signal. The signal ranges between the values -2 and +2. There are 4 bits or 16 levels (hence 16 code numbers) available. The levels assigned have symmetry like we demonstrated in class. The first three sample values obtained (before quantization) are 0.9, 2.0, and -1.4, respectively.

a. Find the quantized values for the three sample values.

b. Find the corresponding PCM sequences for the quantized values.

Solution:

Bin Interval	Quantized Value	PCM Codeword
[-2, -1.75)	-1.875	0000
[-1.75, -1.5)	-1.625	0001
[-1.5, -1.25)	-1.375	0010
[-1.25, -1)	-1.125	0011
[-1, -0.75)	-0.875	0100
[-0.75, -0.5)	-0.625	0101
[-0.5, -0.25)	-0.375	0110
[-0.25, 0)	-0.125	0111
[0, 0.25)	0.125	1000
[0.25, 0.5)	0.375	1001
[0.5, 0.75)	0.625	1010
[0.75, 1)	0.875	1011
[1, 1.25)	1.125	1100
[1.25, 1.5)	1.375	1101
[1.5, 1.75)	1.625	1110
[1.75, 2]	1.875	1111

Table 1: PCM Table

a. Given the sample values $\{0.9, 2.0, -1.4\}$, we can find the quantized values with the above table as $\{0.875, 1.875, -1.375\}$.

b. The corresponding PCM sequence is $\{1011, 1111, 0010\}$.

Problem 6. Let $s(t) = 10\cos(2\pi ft + \pi/4)$ where $f = 20$ Hz. Let us sample $s(t)$ at the sampling rate of $f_s = 80$ Hz to obtain the discrete time signal $s(nT_s) = 10\cos(2\pi fnT_s + \pi/4)$ where $T_s = 1/f_s$, for $n = 0, 1, 2, \ldots, 40$. Using the PCM example in class as a guide compute the quantized PAM signal and the corresponding PCM codeword assuming you have 8 bits or 256 levels to represent the quantized signal.

Note: In this problem you are to use Matlab.

Solution:

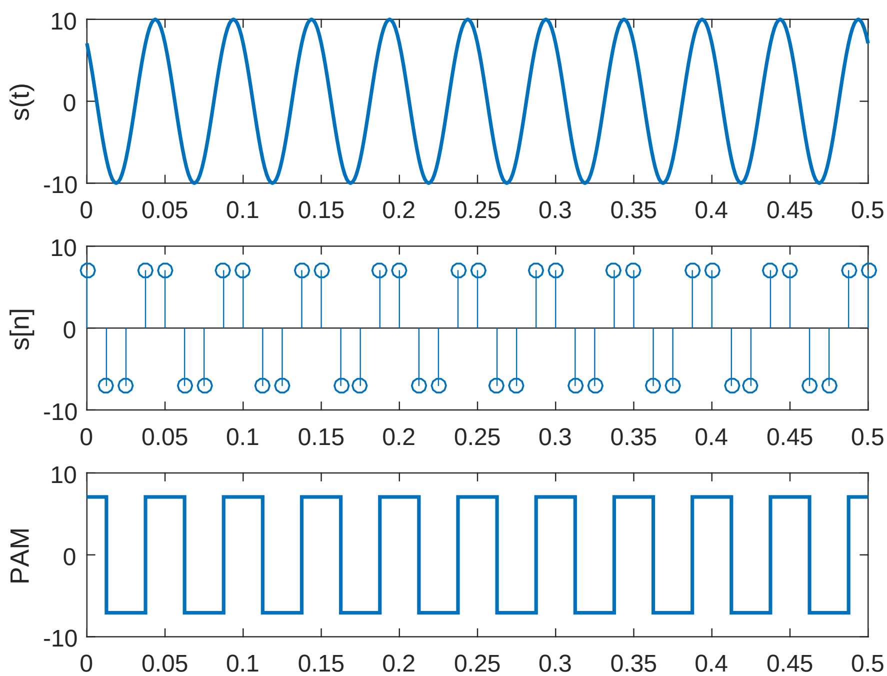

Figure 1: Original signal $s(t)$, its discrete-time sample $s[n]$ with $f_s = 80$Hz and the corresponding PAM signal.

Observe that after sampling $s(t)$ with sampling rate $f_s = 80$ Hz, we get a discrete signal $s[n]$ with only two values, 7.0711 and -7.0711 which correspond to code number 218 (quantized value 7.0703) and code number 37 (quantized value -7.0703), respectively.

Hence, the PCM sequence is $\{218, 37, 37, 218, \cdots\}$ or with binary representation $\{11011010, 00100101, 00100101, 11011010, \cdots\}$.