EE 567

Homework 11

(Not to be handed in)

Work all 5 problems.

Problem 1 (from EE 567 Final, Fall 2014). A binary communication system is used where at the receiver the test statistic is $z(T) = ca_i + n_0,\ i = 1, 2$ where $a_1 = +1$ and $a_2 = -1$ and $c = 1$ if a signal is present and $c = 0$ if a signal is not present. The noise n_0 is distributed as

$$p(n_0) = \begin{cases} n_0 + 1.0, \ -1.0 \leq n_0 \leq 0 \\ \\ -(n_0 - 1.0), \ 0 \leq n_0 \leq 1.0 \\ \\ 0 \text{ elsewhere.} \end{cases}$$

However, the receiver is just interested in detecting whether or not the signal is actually present. If the signal is not present the noise n_0 is still present and if the signal is present the noise is present also. So, the receiver designer decides to take the absolute value of the test statistic and compare this to a threshold. If $|Z(T)| > T_0$, for some T_0, then a signal is declared present, otherwise, it is not.

a. Find T_0 such that the probability of false alarm is 10^{-4}.

b. If a signal is present find the probability that you would detect it with this logic if the value of $T_0 = 3/4$ (which is not the answer to part (a)).

Problem 2 (from EE 567 Final, Fall 2014). Explain the concept of M of N detection. How does this compare (in terms of dB performance and implementation complexity) to the usual integration detection? Also, give the formula you should use to calculate the overall probability of detection given that the output of the energy detector before the M of N logic exceeds the detection threshold with probability $p_{d,s}$. For a given value of N what is a good value of M to use?

Problem 3 (revised from EE 567 Final, Fall 2014). Suppose we receive a BPSK signal of the form

$$r(t) = A_1 \cos(2\pi f_c t) + n(t), \quad 0 \le t \le T$$

where $n(t)$ is a noise process, $T = 1$ sec, $f_c = 1$ kHz and $A_1 = \pm 2$ with equal probability. The signal is demodulated in an optimal manner and then an optimal threshold is used in making a decision. For demodulation assume the received signal $r(t)$ is mixed with a cosine wave of amplitude 1 with the same frequency and phase of the transmitted signal of interest and then low pass filtered by integrating from 0 to T. With this approach suppose at the output of the demodulator (just prior to the threshold comparator) the noise (call it n) is characterized by the Laplacian density

$$p(n) = \frac{1}{\sqrt{2}\sigma} e^{-|n|\sqrt{2}/\sigma}, \quad (-\infty < n < \infty)$$

where we will take $\sigma = 1$.

a. Using the optimal detector (with the corresponding optimal threshold) as described above compute the probability of a correct decision, P_c, for the value of A_1. You may assume that f_c is known to the receiver.

b. Now suppose an interfering signal is also present so that

$$r(t) = A_1 \cos(2\pi f_c t) + A_2 \cos(2\pi f_c t + \pi/4) + n(t) \quad 0 \le t \le T.$$

However, the receiver does not know the interfering signal is present so the receiver does not change the threshold used in part (a) nor does the receiver make any other modifications. Find the value of A_2 so that if a symbol is transmitted with $A_1 = 2$ then the probability of a correct decision for that symbol is now 0.1.

c. Using the value of A_2 found in part (b) find the probability of a correct decision at the receiver assuming $A_1 = -2$.

d. Combine parts (b) and (c) to compute the overall probability of a correct decision in the presence of the interferer.

e. Again using the value of A_2 found in part (b) find the overall probability of a correct decision if the interferer's frequency is changed such that now $f_0 = 0.8$ kHz and thus

$$r(t) = A_1 \cos(2\pi f_c t) + A_2 \cos(2\pi f_0 t + \pi/4) + n(t) \quad 0 \le t \le T.$$

Problem 4 (from Sklar, Digital Communications, 2nd ed.). Suppose an airplane terminal communicating with a satellite is equipped with a frequency hopping spread spectrum system transmitting with an EIRP of 20 dBW (dB Watts). The data rate is $R = 100$ bits/sec. The jammer is transmitting wideband Gaussian noise continually with $\text{EIRP}_J = 60$ dBW. Assume that $(E_b/J_0)_{reqd} = 10$ dB and that the path loss is identical for both the airplane terminal and the jammer.

a. Should the communicators be more concerned with jammer trying to jam the uplink or the downlink?

b. If it is desired to have an AJ margin of 20 dB, what should be the value of the hopping bandwidth W_{ss}?

Problem 5 (from Sklar, Digital Communications, 2nd ed.). Assume that a repeat-back jammer is located $d = 30$ km away from the communicator. Assume further that the jammer can monitor any uplink transmission from the communicator to a nearby satellite. How fast must the communicator hop his frequency to evade the repeat-back jammer? Assume that the jammer can change its jamming frequnecy in zero time and that the only differential delay between the communicator's uplink signal and the jamming uplink signal is the propogation delay from the communicator to the jammer.