EE 484

Homework 4 Solutions

Problem 1. A BPSK signal is utilized at 10,000 bits/sec. The equally likely waveforms are $s_1(t) = A\cos(\omega_0 t)$ and $s_2(t) = -A\cos(\omega_0 t)$, where $A = 1$ mV and the single-sided noise density $N_0 = 5 \times 10^{-12}$ W/Hz. Assume the signal power and enery per bit are normalized to a 1-Ω resistive load. Find the expected number of bit errors made in one hour at the receiver.

Solution:

$$E_b = \frac{A^2 T}{2} = \frac{(1 \times 10^{-3})^2 \times 10^{-4}}{2} = 5.0 \times 10^{-11}.$$

$$P_b = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) = Q\left(\sqrt{\frac{2 \times 5.0 \times 10^{-11}}{5 \times 10^{-12}}}\right) = Q(\sqrt{20}) = 3.87 \times 10^{-6}.$$

The expected number of bit errors in one hours is

$$N_b = 3.87 \times 10^{-6} \times 10,000 \times 3600 = 139.4.$$

Problem 2. Suppose we have BPSK modulation and we implement an error correction code, say, a (15,5) BCH code ($n = 15,\ k = 5$). This code can correct 3 bit errors in a block of size 15 bits. Plot on the same graph P_b vs. E_b/N_0 for both the uncoded and coded waveform. Your E_b/N_0 values should be in dB on the plot. Your plot should be done using a software package such as Excel or Matlab.

Solution: For the uncoded case we have

$$P_{b,uc} = Q\left(\sqrt{\frac{2E_b}{N_0}}\right).$$

For the coded case we have

$$P_{b,c} = \frac{1}{15}\sum_{k=4}^{15} k \binom{15}{k} P_{b,uc}^k (1 - P_{b,uc})^{15-k}.$$

In plotting the coded case we have to account for the overhead by adjusting the E_b/N_0 (dB) scale by $10\log_{10}\left(\frac{15}{5}\right) = 4.77$ (dB). See the plot below.

Problem 3. Suppose we have a QPSK constellation where each point (or symbol) is at a radius of r_1 from the origin. The performance of this system depends on the distance between the constellation points and is mostly determined by the distance between adjacent points, d. It is a simple application of the Pythagorean theorem to show that $r_1 = d/\sqrt{2}$ (verify this). With this system we transmit 2 bits per constellation symbol. Suppose now we consider an 8PSK constellation where we transmit 3 bits per symbol. If we used the same radius for the points as was used for the QPSK system the points would be closer to each other in distance since we now have 8 points on the ring instead of 4. Thus, the probability of a symbol decision error at the receiver is higher for this design. Therefore, if we wish to maintain the same probability of symbol error for the 8PSK system that we had for the QPSK system we have to increase the radius of the ring to some value r_2 so that the distance between adjacent constellation symbols for the 8PSK system is also d.

a. Find the required value of the radius r_2 in terms of d.

   Solution: First we see that for QPSK using the Pythagorean theorem

   $$r_1^2 + r_1^2 = d^2 \Rightarrow r_1 = \frac{d}{\sqrt{2}}.$$

   For 8PSK we have using the law of cosines

   $$d^2 = r_2^2 + r_2^2 - 2r_2r_2\cos\left(\frac{\pi}{4}\right) \Rightarrow r_2 = \frac{d}{\sqrt{2-\sqrt{2}}}.$$

b. How much more power is required for the 8PSK system with a radius of r_2 to achieve the same symbol error performance as the QPSK system when just considering adjacent symbol errors, that is, errors caused by transmitted symbols received closer to adjacent symbols?

   Solution: The average transmitted power for QPSK is

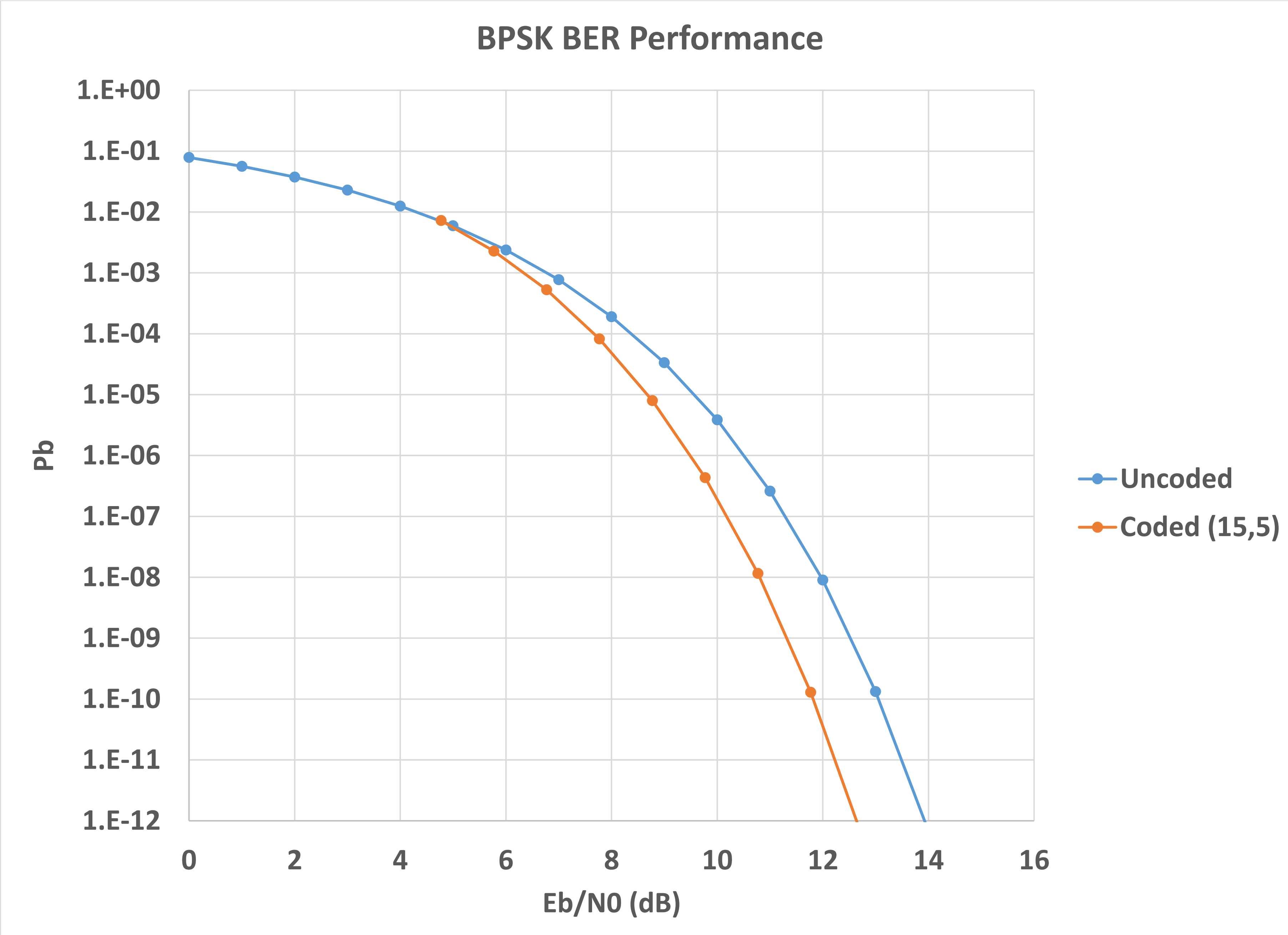
   $$P_{QPSK} = r_1^2 = \frac{d^2}{2}$$

   and the average transmitted power for 8PSK is

   $$r_2^2 = \frac{d^2}{2-\sqrt{2}}.$$

Thus, the additional power is

$$P = 10\log_{10}\frac{2d^2}{(2-\sqrt{2})d^2} = 5.33 \text{ dB}.$$



BPSK BER Performance
1.E+00
1.E-01
1.E-02
1.E-03
1.E-04
1.E-05
1.E-06
1.E-07
1.E-08
1.E-09
1.E-10
1.E-11
1.E-12
Pb
0
2
4
6
8
10
12
14
16
Eb/N0 (dB)
Uncoded
Coded (15,5)