EE 484

Homework 3 Solutions

Problem 1. Recall that for BPSK signaling the probability of a bit error is

$$P_b = Q\left(\sqrt{\frac{2E_b}{N_0}}\right).$$

Now suppose we implement an error correction code, say, a (7,4) Hamming code ($n = 7,\ k = 4$). This code can correct 1 bit error in a block of size 7 bits. Plot on the same graph P_b vs. E_b/N_0 for both the uncoded and coded waveform. Remember to account for the coding overhead in your plot since E_b/N_0 applies to information bits. Your E_b/N_0 values should be in dB on the plot.

Solution: For the coded case let $p = P_b$ where P_b is as defined above. Then the coded P_b is

$$P_b = \frac{1}{7}\sum_{j=2}^{7} j\binom{n}{j}p^j(1-p)^{7-j}.$$

The plots are shown below. Note that for the coded case the uncoded Eb/N_0 is shifted to the right by $10 \times \log_{10}\left(\frac{7}{4}\right)$.

Problem 2. A receiver front end has a noise figure of 10 dB and a gain of 60 dB and a bandwidth of 8 MHz. The input signal power is 10^{-11} W. The antenna temperature is 170 K. Find $T_e,\ T_s,\ N_{out},\ SNR_{in}$ and SNR_{out}.

Solution:

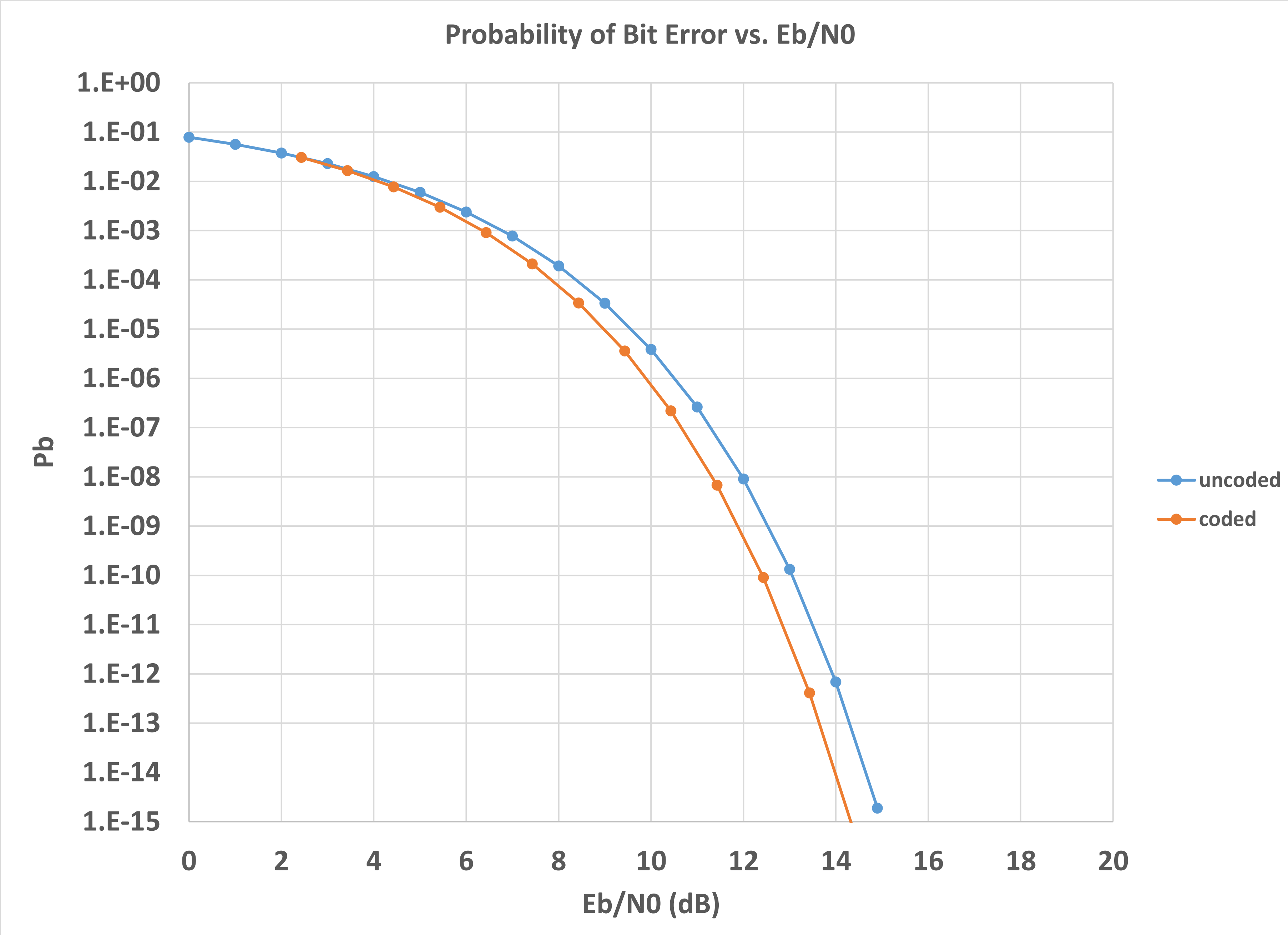
$$T_e = (F-1)290 \text{ K} = (10^{(10/10)} - 1) \times 290 \text{ K} = 2610 \text{ K}$$

$$T_s = T_a + T_e = 170 \text{ K} + 2610 \text{ K} = 2780 \text{ K}$$

$$N_{out} = GKT_sB = 10^{60/10} \times 1.38 \times 10^{-23} \times 2780 \times 8 \times 10^6 = 3.07 \times 10^{-7}$$

$$SNR_{in} = \frac{S_{in}}{kT_aB} = \frac{10^{-11}}{1.38 \times 10^{-23} \times 170 \times 8 \times 10^6} = 532.82 = 27.27 \text{ dB}$$

$$SNR_{out} = \frac{S_{out}}{N_{out}} = \frac{S_{in}G}{N_{out}} = \frac{10^{-11} \times 10^{60/10}}{3.07 \times 10^{-7}} = 32.58 = 15.13 \text{ dB}$$


Probability of Bit Error vs. Eb/N0
1.E+00
1.E-01
1.E-02
1.E-03
1.E-04
1.E-05
1.E-06
1.E-07
1.E-08
1.E-09
1.E-10
1.E-11
1.E-12
1.E-13
1.E-14
1.E-15
Pb
0
2
4
6
8
10
12
14
16
18
20
Eb/N0 (dB)
uncoded
coded