EE 484
Homework 1 Solutions

Problem 1. Explain the meaning of negative frequency.

Solution: Consider

$$e^{\pm j\omega_0 t} = \cos(\omega_0 t) \pm j\sin(\omega_0 t).$$

The rotation rate for both complex sinusoids $e^{\pm j\omega_0 t}$ is $|\omega_0|$. The positive sign means the rotation is counterclockwise and the negative sign means the rotation is clockwise.

Problem 2. Determine for each of the following whether or not the discrete-time system is linear, and/or time-invariant.

a. $y(n) = 2\cos[x(n) + 2]$.

Solution: Let

$$x(n) = ax_1(n) + bx_2(n).$$

Then

$$\begin{aligned} y(n) &= 2\cos[ax_1(n) + bx_2(n) + 2] \\ &\neq ay_1(n) + by_2(n) = a2\cos[x_1(n) + 2] + b2\cos[x_2(n) + 2] \end{aligned}$$

so the system is not linear.

Let $x(n) = x_1(n-k)$. Then

$$\begin{aligned} y(n) &= 2\cos[x_1(n-k) + 2] \\ &= y_1(n-k) \end{aligned}$$

so it is time (or shift) invariant.

b. $y(n) = x(n)$.

Solution: linear, time invariant.

c. $y(n) = \log_{10}(|x(n)|), \quad x(n) \neq 0$.

Solution: not linear, time invariant.

d. $y(n) = x(n) + n$.

Solution: linear, not time invariant.

Problem 3. Compute the Fourier transform of the triangular pulse, $Tri(t)$, where

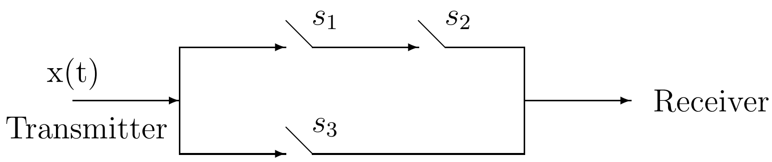
$$Tri(t) = \begin{cases} t+1, & -1 \leq t \leq 0, \\ 1-t, & 0 < t \leq 1, \\ 0, & elsewhere \end{cases}$$

and sketch a plot of its graph in the frequency domain. You should produce two plots (one for the magnitude of the Fourier transform and one for the phase). In the magnitude plot, clearly identify where the maximum height occurs and the value of the maximum height. Also, identify where the approximate zero-crossings occur (in doing this part you can ignore the contributions to the zero crossings due to that part of the signal whose energy is mostly concentrated away from the zero crossing you are identifying).

Hint: Think of the triangular pulse as the convolution of two rectangular pulses and use a Fourier transform property.

Solution: This function is the convolution of a rectangular pulse with itself where the rectangular pulse extends from -1/2 to +1/2 and has a height of 1. The Fourier transform of the rectangular pulse is $sinc(f)$ so the Fourier transform of $Tri(t)$ is $sinc^2(f)$ and the zero crossings occur when $\sin(\pi f) = 0$ (except when $f = 0$) so they occur at all non-zero integer values.

Problem 4. Consider the transmission of a signal as shown in the following diagram.



A signal is transmitted along two paths as shown. In the upper path there are two switches to pass through while in the lower path there is one switch to pass through. Each switch s_i operates independently and allows the signal to pass with probability p_i for $i = 1, 2, 3$. The signal transmission is successful if the signal $x(t)$ sent at the transmitter reaches the receiver along either or both paths. Find the probability that the transmission is successful if $p_1 = 1/2,\ p_2 = 1/3,\ p_3 = 1/4$.

Solution. Let U be the event of the signal passing thru the upper path and let L be the event of the signal passing thru the lower path. Then,

$$P(\text{success}) = P(U \cup L) = P(U) + P(L) - P(U \cap L)$$

so

$$P(\text{success}) = \frac{1}{2} \cdot \frac{1}{3} + \frac{1}{4} - \frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{4} = \frac{3}{8}.$$

Problem 5. Suppose the random variable X has mean 2 and variance 6. Let $Y = X^2 + 1$. Find the mean of Y.

Solution:

$$E[Y] = E[X^2 + 1] = Var(X) + (E[X])^2 + 1 = 6 + 4 + 1 = 11.$$

Problem 6. Suppose the normal random variable X has mean 2 and variance 6. Let $Y = X^2 + 1$. Find the variance of Y.

Solution:

$$E[Y] = 11.$$

$$E[Y^2] = E[X^4 + 2X^2 + 1].$$

First let

$$Z = \frac{X - E[X]}{\sqrt{Var(X)}} = \frac{X - \mu_X}{\sigma_X}.$$

Then Z is a standard normal random variable with mean 0 and variance 1. Note that $E[Z^4] = 3$. Now

$$X = \mu_X + \sigma_X Z$$

so

$$X^4 = \mu_X^4 + 4\mu_X^3\sigma_X Z + 6\mu_X^2\sigma_X^2 Z^2 + 4\mu_X\sigma_X^3 Z^3 + \sigma_X^4 Z^4.$$

Hence,

$$E[X^4] = \mu_X^4 + 6\mu_X^2\sigma_X^2 + 3\sigma_X^4$$

or

$$E[X^4] = 16 + 144 + 108 = 268.$$

Also,

$$E[X^2] = Var(X) + (E[X])^2 = 6 + 4 = 10.$$

Thus,

$$Var(Y) = E[Y^2] - (E[Y])^2 = 268 + 20 + 1 - 121 = 168.$$