EE 484

Raised-Cosine Filter Notes

Reference: B. Sklar, *Digital Communications*, 2nd ed., Prentice Hall, 2001.

A frequently used transfer function for zero intersymbol interference at sampling times is the raised-cosine filter

$$H(f) = \begin{cases} 1, & |f| \le 2W_0 - W \\ \cos^2\left(\dfrac{\pi}{4}\dfrac{|f| + W - 2W_0}{W - W_0}\right), & 2W_0 - W < |f| < W \\ 0, & |f| > W \end{cases}$$

where W is the absolute bandwidth ad $W_0 = 1/2T$ is the minimum Nyquist bandwidth for the rectangular spectrum and -6 dB bandwidth for the raised-cosine spectrum. The system impulse is

$$h(t) = 2W_0(\text{sinc } 2W_0 t)\frac{\cos[2\pi(W - W_0)t]}{1 - [4(W - W_0)t]^2}.$$

The difference $W - W_0$ is called the excess bandwidth which means the additional bandwidth beyond the Nyquist minimum (for a rectangular system $W = W_0$). The roll-off factor r is defined as

$$r = \frac{W - W_0}{W_0}, \quad 0 \le r \le 1.$$

Thus, r characterizes the steepness of the filter roll off. Note that $r = 0$ is the Nyquist minimum case and when $r = 1$ the excess bandwidth is 100% and the tails of the system impulse response are small. A system with this overall spectral characteristic can provide a symbol rate of R_s symbols/sec using a bandwidth of R_s Hz (twice the Nyquist minimum bandwidth), thus yielding 1 symbol/sec/Hz. The raised-cosine filter is not physically realizable (it is noncausal) but can be approximately implemented. Note that the raised-cosine filter is the composite filter for both the transmitter and receiver. Often this is achieved by utilizing matched root-raise cosine filters (square root of the raised cosine) at the transmitter and receiver so that the product of the two filters is the raised-cosine filter.

Note that the larger the filter roll-off the shorter will be the pulse tails and

also smaller tail amplitudes which results in less sensitivity to timing errors and thus smaller degradations due to ISI. The cost for this is more excess bandwidth. Thus the smaller the filter roll-off the smaller will be the excess bandwidth allowing an increase in signal rate or the number of users that can use the system at the same time. But the smaller filter roll-off results in more ISI. So there is a trade here.