# EE 484

**Midterm Exam**

Due Monday, March 28, 2016 at 6:30 p.m. in class

**Work all 9 problems.**

**Problem 1.** [12 points]. Suppose $X_1, X_2, \ldots$ are each independent and normally distributed with mean zero and variance one (standard normal). Define $Y_1 = X_1$ and for $n = 2, 3, 4, \ldots$ let

$$Y_n = \alpha X_n + (1 - \alpha) Y_{n-1}, \quad 0 < \alpha < 1.$$

a. Compute $E[Y_n Y_m]$ as a closed form function of $n$, $m$ and $\alpha$ for $n \geq m$. Simplify as much as possible.

b. Evaluate your expression for $E[Y_n Y_m]$ using $n = 10$, $m = 5$ and $\alpha = 0.2$.

**Problem 2.** [10 points]. Consider a real Gaussian random sequence $x(n)$, $n$ an integer, with

$$E[x(n)] = 0, \quad E\left[x(n)^2\right] = 1, \quad E[x(n)x(m)] = \rho^{|n-m|}$$

where $0 < \rho < 1$. Let

$$y(n) = 2x(n) + 2.$$

a. Is $x(n)$ wide sense stationary?

b. Find the covariance of $y(n)$ and state whether or not it is wide sense stationary.

**Problem 3.** [10 points]. Consider a wide-sense stationary random process $X(t)$ with autocorrelation function

$$R_X(\tau) = \begin{cases} 1 - \dfrac{|\tau|}{T}, & |\tau| \leq T \\ 0, & |\tau| > T. \end{cases}$$

Find $S_X(f)$, the power spectral density of $X(t)$.

**Problem 4.** [6 points]. What are the two primary sources of error that lead to pointing losses by an antenna?

**Problem 5.** [6 points]. Suppose a certain system that utilizes an A/D converter has 7 effective bits. It has been determined that 8 effective bits are needed. One could use another A/D coverter that has more physical bits. However, if that is not possible one can require that SINAD increase. How much would SINAD have to increase to obtain 8 effective bits instead of the current 7 effective bits?

**Problem 6.** [16 points]. Consider a signal detector with an input

$$r = \pm A + n$$

where $+A$ and $-A$ occur with equal probability and the noise variance $n$ is characterized by the Laplacian density

$$p(n) = \frac{1}{\sqrt{2}\sigma} e^{-|n|\sqrt{2}/\sigma}.$$

a. Derive the probability of error as a function of $A$ and $\sigma$.

b. Find the SNR ($= A^2/\sigma^2$) required to achieve an error probability of $10^{-5}$.

c. Repeat part (a) for the Gaussian density

$$p(n) = \frac{1}{\sqrt{2\pi}\sigma} e^{-n^2/2\sigma^2}.$$

d. Repeat part (b) for the above Gaussian density and compare the results you got using the Laplacian and Gaussian densities.

**Problem 7.** [10 points]. A receiver front end has a noise figure of 8 dB and a gain of 60 dB and a bandwidth of 7 MHz. The input signal power is $10^{-11}$ W. The antenna temperature is 165 K.

a. Find $T_e$, $T_s$, $N_{out}$, $SNR_{in}$ and $SNR_{out}$..

b. Repeat part [a] but using an antenna temperature of 8000 K (the sun is in the field of view).

**Problem 8.** [12 points]. Suppose we have BPSK modulation and we implement an error correction code, say, a (15,7) binary BCH code ($n = 15,\ k = 7$). This code can correct 2 bit errors in a block of size 15 bits. Plot on the same graph $P_b$ vs. $E_b/N_0$ for both the uncoded and coded waveform. Your $E_b/N_0$ values should be in dB on the plot and should cover a range starting at 0 dB and extending far enough so that $P_b = 10^{-12}$ appears on the graph. Your plot should be done using a software package such as Excel or Matlab. Your $P_b$ values should be plotted using a log scale.

**Problem 9.** [18 points]. Suppose we have an 8-bit A/D converter. Let us number these 256 levels from 1 to 256. Then the 0 voltage level lies halfway between levels 128 and 129. Suppose we have an input to the A/D consisting of two sinusoidal signals and noise of the form

$$s(t) = A_1 \cos(2\pi f_1 t) + A_2 \cos(2\pi f_2 t) + n(t)$$

where $n(t)$ is AWGN. Suppose that $A_1 = 10$, $A_2 = 5$, $f_1 = 2000$ Hz and $f_2 = 3000$ Hz. Assume that at any particular sample time $n(t)$ is mean 0 with variance $\sigma^2$ and is independent from sample to sample. Suppose each level of the A/D corresponds to a 1 volt change. Say we have a gain stage at the input to the A/D that we can apply to $s(t)$ to form $cs(t)$. We would like to find the maximum value of the constant $c$ so that the output of the A/D from time 0 to infinity will clip no more than one percent of the time.

Note that this problem is *not* asking that the probability of clipping at the peak of $s(t)$ is 0.01. Rather, it is asking for the maximum value of $c$ so that the output observed over a long time will clip no more than one percent of the time.

a. Using a sample rate of $f_s$ Hz, give an analytical expression that one would have to evaluate to determine if a particular $c$ would work, in general.

b. Suppose the composite signal $s(t)$ is sampled at 15000 Hz and the noise variance is $\sigma^2 = 1$. Find the numerical value of $c$ required in the problem.

c. Suppose the composite signal $s(t)$ is sampled at 150000 Hz and the noise variance is $\sigma^2 = 1$. Find the numerical value of $c$ required in the problem.

**Note**: If you use Matlab to aid you in part [b] and/or part [c] remember to hand in your code. Your Matlab code should only be used to evaluate analytic expressions (possibly iteratively until the correct $c$ is found), not to perform a simulation where you simulate $s(t)$. Credit will not be given for answers obtained via a simulation of $s(t)$.