EE 484

Homework 7 Solutions

Problem 1. This is a Matlab exercise. Analog to digital converters are used in most digital communication systems today. These devices accept analog signals as input, quantize them at a defined sample rate, and assign a set of bits (a symbol) to each sample according to the amplitude. For example, an ideal n-bit converter will quantize input signals into one of 2^n levels. For this exercise, assume that the quantization steps are uniform in amplitude.

a. Generate the signal $s(n) = A \cdot sin(2\pi f_c \Delta t \cdot n)$, for $f_c = 2$, A=1.0, $\Delta t = 0.005$, and $1 \leq n \leq 20,000$. Assume there are four physical bits of resolution in the ADC, then generate and plot the first 200 samples of the input waveform and the quantized waveform (together on the same plot), using a "mid-point" quantizer, so that input values are mapped to the midpoint of the corresponding bin.

b. Generate and plot the error signal between the quantized signal and the input signal. Numerically calculate the mean and variance of this signal, and compare to the theoretical values discussed in class.

c. Generate and plot the quantized waveform for five and six bits of resolution in the analog-to-digital converter, along with the mean and variance of the error signals, and compare to the theoretical values.

d. Calculate the signal to noise ratio for the four, five and six bit cases. What effect do you think non-uniformities in the step sizes have on the signal-to-noise ratio?

Solution: See plot below. Also see example code below.

```
% Quantization of signals. HW7 problem 1 for EE484

A = 1.0;                                          % Amplitude of modulating waveform (divide by A to get
f_c = 2;                                          % Carrier frequency
delta_t = 0.005;                                  % Sample step size in time
phi = pi/2;                                       % Message phase
N = 20000;                                        % Number of time samples
numBits = 4.0;                                    % Number of bits in the quantizer
minValue = -A;                                    % Min level in the quanitzer
maxValue = +A;                                    % Max value in the quantizer

t = [1:N] .* delta_t;                             % Create vector of time samples

numLevels = 2^numBits;                            % Number of levels in the quantizer
stepSize = (maxValue - minValue)/numLevels;       % Corresponding step size

Q = zeros(1, numLevels);

for q = 1:numLevels                               % Set the quantization levels
    Q(q) = minValue + (q-1)*stepSize + stepSize/2;
end;

s = A * sin(2.0 * pi * f_c * t + phi);            % Create the signal to be quantized
```

```
plot(s(1:200)); hold on;

for n=1:N                                           % Create the quantized waveform
    if (s(n) <= Q(1)+stepSize/2)
        s_q(n) = Q(1);
    else
        if (s(n) >= Q(numLevels)+stepSize/2)
          s_q(n) = Q(numLevels);
        else
            q=1;
            while (s(n) > Q(q)+stepSize/2)
                q=q+1;
            end
            s_q(n) = Q(q);
        end
    end
end

plot(s_q(1:100), '-g'); hold off;
xlabel('Samples');
ylabel('Amplitude');
str = sprintf('Quantized Signal (%d bits)', numBits);
legend('Input Signal', str); hold off; figure;

error = s - s_q;                                    % Generate the error signal
meanError = mean(error);                            % Calculate the mean of the error signal
varError = var(error);                              % Calculate the variance of the error signal
predError = stepSize^2/12;                          % Calculate the predicted variance for comparison

plot(error(1:200));
str = sprintf('Error Signal (%d bits)', numBits);
legend(str);
```

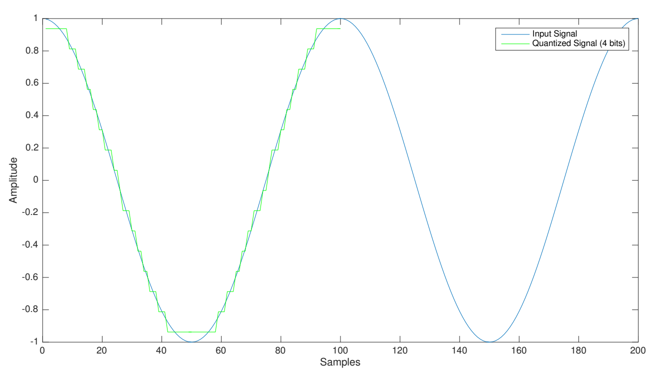

Figure 1: Plot for problem 1a - 4 bits of ADC resolution

Generate and plot the error signal between the quantized signal and the input signal. Numerically calculate the mean and variance of this signal, and compare to the theoretical values discussed in class.

Solution: See plot below. For this example, the mean is -1.9e-05 (note that this is a function of selecting the carrier phase to be $\pi/2$) and the variance is 1.6e-3. Theoretical values are zero for the mean and 1.3e-3 for the variance ($\sigma^2 = \Delta^2/12$).

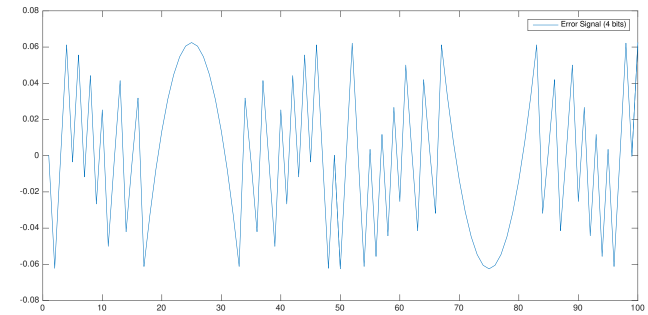

Figure 2: Plot for problem 1b - error signal resulting from quantization

Generate and plot the quantized waveform for four bits and five bits of resolution in the analog-to-digital converter, along with the mean and variance of the error signals, and compare to the theoretical values.

Solution: See plot below. For the five bit case, the mean is -9.4e-06 (again note that this is a function of selecting the carrier phase to be $\pi/2$) and the variance is 4.9e-04. Theoretical values are zero for the mean and 3.3e-4 for the variance. For the six bit case, the mean is -4.7e-06 (and again note that this is a function of selecting the carrier phase to be $\pi/2$) and the variance is 1.3e-04. Theoretical values are zero for the mean and 8.1e-5 for the variance.

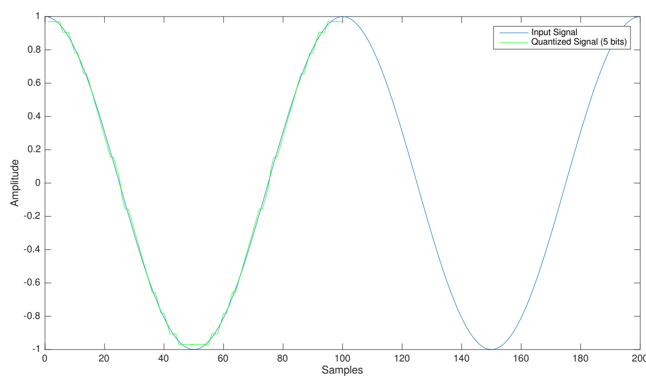

Figure 3: Plot for problem 1c (5 bits of ADC resolution)

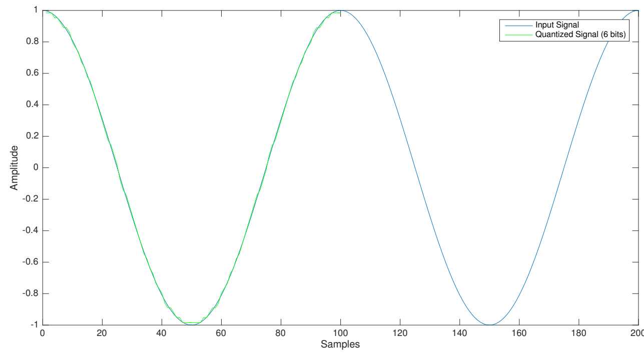

Figure 4: Plot for problem 1c (6 bits of ADC resolution)

Calculate the signal to noise ratio for the four, five and six bit cases. What effect do you think non-uniformities in the step sizes have on the signal-to-noise ratio?

Solution: From the lectures, we know that the ideal SNR (or more precisely, the signal to quantization noise) for a uniform quantizer can be given as $1.761 + 6.02N$ dB, where n is the number of bits. From this equation, the ideal SNR for a four bit quantizer is 25.84 dB. The ideal SNR for a five bit uniform quantizer is 31.86 dB, and the ideal SNR for a six bit uniform quantizer is 37.88 dB. In practice, the quantization process is not perfect, and variations in the levels will occur. This additional variation reduces the SNR from the ideal values described by the equation above, resulting in an effective number of bits (ENOB) for the ADC that is something less than the ideal number, as described in class.

Problem 2. A receiver front end has a noise figure of 8 dB and a gain of 60 dB and a bandwidth of 5 MHz. The input signal power is 10^{-11} W. The antenna temperature is 175 K. Find $T_e,\ T_s,\ N_{out},\ SNR_{in}$ and SNR_{out}. Recall that $T_0 = 290$ K.

Solution:

$$T_e = (F-1)290 \text{ K} = (10^{(8/10)} - 1) \times 290 \text{ K} = 1540 \text{ K}$$

$$T_s = T_a + T_e = 175 \text{ K} + 1540 \text{ K} = 1715 \text{ K}$$

$$N_{out} = GKT_sB = 10^{60/10} \times 1.38 \times 10^{-23} \times 1715 \times 5 \times 10^6 = 1.18 \times 10^{-7}$$

$$SNR_{in} = \frac{S_{in}}{kT_aB} = \frac{10^{-11}}{1.38 \times 10^{-23} \times 175 \times 5 \times 10^6} = 828.16 = 29.18 \text{ dB}$$

$$SNR_{out} = \frac{S_{out}}{N_{out}} = \frac{S_{in}G}{N_{out}} = \frac{10^{-11} \times 10^{60/10}}{1.18 \times 10^{-7}} = 84.52 = 19.27 \text{ dB}$$

Problem 3. Using the same design as Problem 2 an additional amplifier is inserted in the system before the one described in Problem 1 (a preamplifier) so that now the antenna feeds energy to two networks in cascade. The preamp has a noise figure of 3 dB and a gain of 13 dB and a bandwidth of 5 MHz. Find $T_s,\ F_{out},\ N_{out}$ and SNR_{out}, where F_{out} is the overall or composite F.

Solution:

$$T_{e1} = (F_1 - 1)290 \text{ K} = (10^{3/10} - 1) \times 290 \text{ K} = 288.63 \text{ K}$$

$$T_{e2} = (F_2 - 1)290 \text{ K} = (10^{8/10} - 1) \times 290 \text{ K} = 1539.78 \text{ K}$$

$$T_{out} = T_{e1} + \frac{T_{e2}}{G_1} = 288.63 + \frac{1539.78}{10^{13/10}} = 365.80 \text{ K}$$

$$T_s = T_a + T_{out} = 175 \text{ K} + 365.80 \text{ K} = 540.80 \text{ K}$$

$$F_{out} = F_1 + \frac{F_2 - 1}{G_1} = 10^{3/10} + \frac{8-1}{10^{13/10}} = 2.26 = 3.54 \text{ dB}$$

$$N_{out} = GKT_sB = 10^{13/10} \times 10^{60/10} \times 1.38 \times 10^{-23} \times 540.80 \times 5 \times 10^6 = 7.45 \times 10^{-7}$$

$$SNR_{out} = \frac{S_{out}}{N_{out}} = \frac{S_{in}G_1G_2}{N_{out}} = \frac{10^{-11} \times 10^{13/10} \times 10^{60/10}}{7.45 \times 10^{-7}} = 267.99 = 24.28 \text{ dB}$$