EE 484

Homework 6 Solutions

Problem 1. Recall that for BPSK signaling the probability of a bit error is

$$P_b = Q\left(\sqrt{\frac{2E_b}{N_0}}\right).$$

Now suppose we implement an error correction code, say, a (7,4) Hamming code ($n = 7,\ k = 4$). This code can correct 1 bit error in a block of size 7 bits. Plot on the same graph P_b vs. E_b/N_0 for both the uncoded and coded waveform. Remember to account for the coding overhead in your plot since E_b/N_0 applies to information bits. Your E_b/N_0 values should be in dB on the graph and should range from 0 to a large enough value so that $P_b \leq 10^{-12}$.

Solution: For the coded case let P_b is as defined above. Then the coded probability of error P_e is

$$P_e = \frac{1}{7}\sum_{j=2}^{7} j\binom{n}{j} P_b^j (1-P_b)^{7-j}.$$

The plots are shown below. Note that for the coded case the uncoded E_b/N_0 is shifted to the right by $10 \times \log_{10}\left(\frac{7}{4}\right)$. As an alternative to doing this shifting one can utilize the following formula for the channel P_b:

$$P_b = Q\left(\sqrt{\frac{2E_b}{N_0}}\right), \text{ where } E_b/N_0 = (E_{b,uncoded}/N_0) \times 4/7$$

and then compute P_e as above but now do not shift the scale.

Problem 2. An uncoded BPSK signal is utilized at 20,000 bits/sec. The equally likely waveforms are $s_1(t) = A\cos(\omega_0 t)$ and $s_2(t) = -A\cos(\omega_0 t)$, where $A = 1$ mV and the single-sided noise density $N_0 = 5 \times 10^{-12}$ W/Hz. Assume the signal power and enery per bit are normalized to a 1-Ω resistive load. Find the expected number of bit errors made in one hour at the receiver.

Solution: We have

$$P_b = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

where

$$E_b = \frac{A^2T}{2} = 2.50 \times 10^{-11}, \text{ and } T = 1/20,000 \text{ sec}$$

so $P_b = 7.83 \times 10^{-4}$. Hence, the expected number of errors made in 1 hour at the receiver is

$$N_E = P_b/T \times 3600 = 56354.$$

Problem 3. A coded BPSK signal is utilized at 20,000 bits/sec. The error correction code is a (7,4) Hamming code ($n = 7,\ k = 4$) that can correct 1 error in a block of 7 bits. The equally likely waveforms are $s_1(t) = A\cos(\omega_0 t)$ and $s_2(t) = -A\cos(\omega_0 t)$, where $A = 1$ mV and the single-sided noise density $N_0 = 5 \times 10^{-12}$ W/Hz. Assume the signal power and enery per bit are normalized to a 1-Ω resistive load. Find the expected number of bit errors made in one hour at the receiver.

Solution: We have

$$P_b = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

where

$$E_b = \frac{A^2T}{2} \times 4/7 = 1.43 \times 10^{-11}, \text{ and } T = 1/20,000 \text{ sec}$$

so $P_b = 8.41 \times 10^{-3}$. Utilizing the formula for P_e given in Problem 1 we find that $P_e = 6 \times 10^{-5}$ so the expected number of errors made in 1 hour at the receiver is

$$N_E = P_e/T = 4320.$$

Problem 4. Suppose we have BPSK modulation and we implement an error correction code, say, a (15,5) BCH code ($n = 15,\ k = 5$). This code can correct 3 bit errors in a block of size 15 bits. Plot on the same graph P_b vs. E_b/N_0 for both the uncoded and coded waveform. Your E_b/N_0 values should be in dB on the plot. Your E_b/N_0 values should be in dB on the graph and should range from 0 to a large enough value so that $P_b \leq 10^{-12}$.

Solution: For the coded case let P_b is as defined in Problem 1. Then the coded probability of error P_e is

$$P_e = \frac{1}{15} \sum_{j=4}^{15} j \binom{n}{j} P_b^j (1 - P_b)^{15-j}.$$

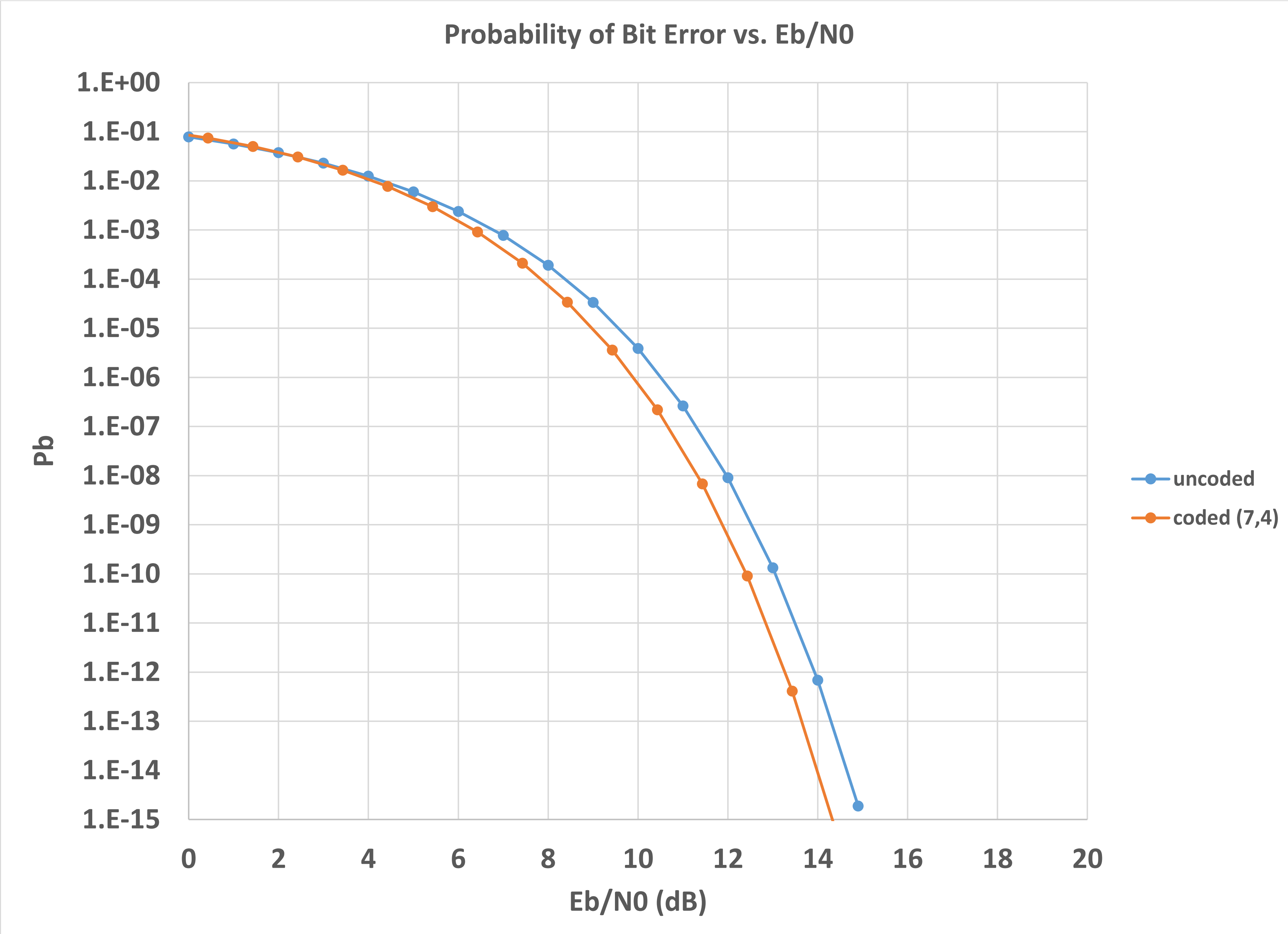
The plots are shown below. Note that for the coded case the uncoded E_b/N_0 is shifted to the right by $10 \times \log_{10} \left(\frac{15}{5}\right)$. As an alternative to doing this shifting one can utilize the following formula for the channel P_b:

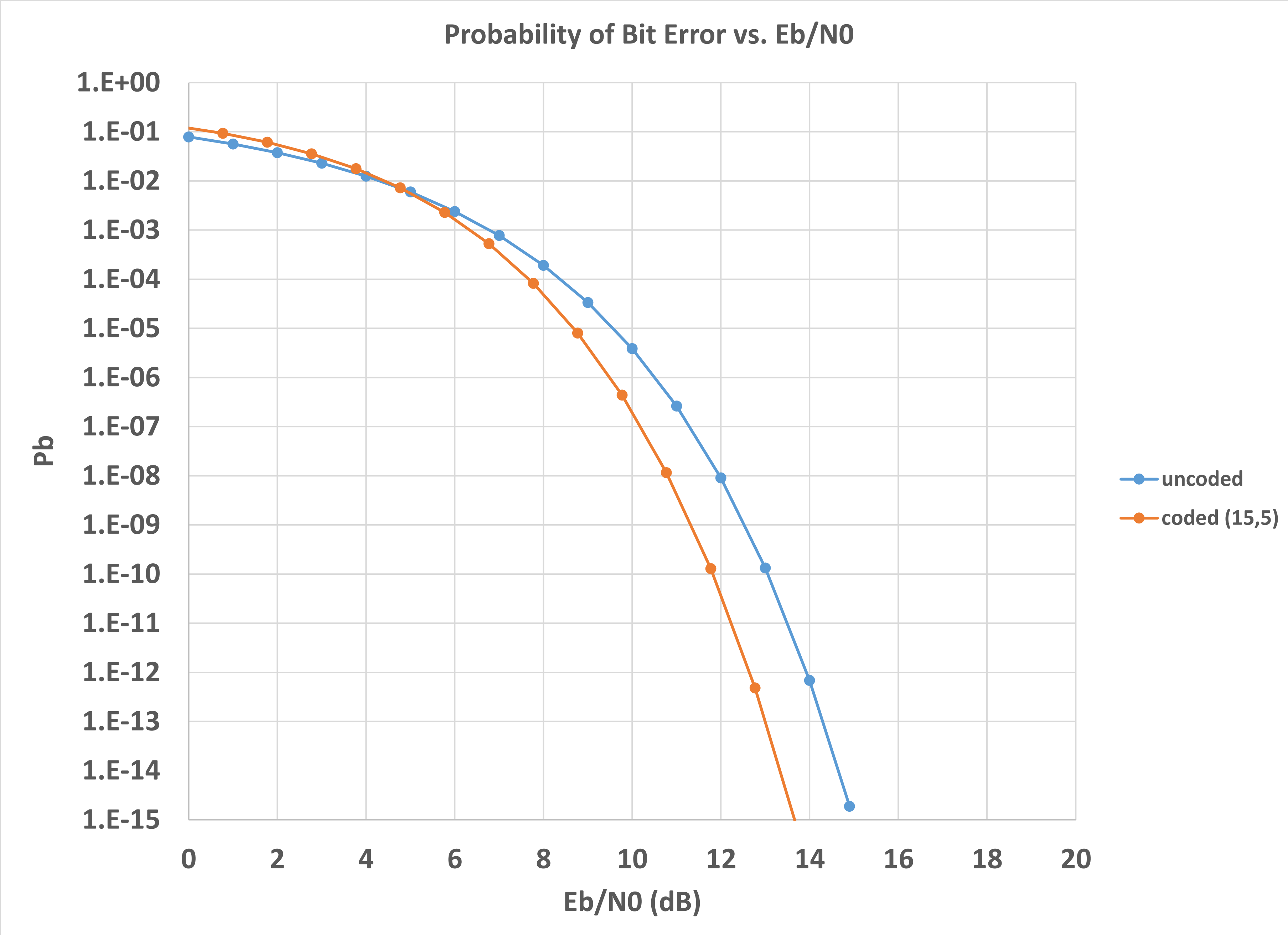
$$P_b = Q\left(\sqrt{\frac{2E_b}{N_0}}\right), \text{ where } E_b/N_0 = E_{b,uncoded}/N_0 \times 5/15$$

and then compute P_e as above but now do not shift the scale.

Problem 5. Matlab exercise. Simulate the signal and error correction code in Problem 1 (uncoded and the coded) and verify that your sim and analytic results match reasonably well by plotting all the curves on the same graph. Your E_b/N_0 values should be in dB on the graph and should range from 0 to a large enough value so that $P_b \leq 10^{-5}$.

Solution: To be posted.


Probability of Bit Error vs. Eb/N0
Pb
1.E+00
1.E-01
1.E-02
1.E-03
1.E-04
1.E-05
1.E-06
1.E-07
1.E-08
1.E-09
1.E-10
1.E-11
1.E-12
1.E-13
1.E-14
1.E-15
0
2
4
6
8
10
12
14
16
18
20
Eb/N0 (dB)
uncoded
coded (7,4)



Probability of Bit Error vs. Eb/N0
1.E+00
1.E-01
1.E-02
1.E-03
1.E-04
1.E-05
1.E-06
1.E-07
1.E-08
1.E-09
1.E-10
1.E-11
1.E-12
1.E-13
1.E-14
1.E-15
Pb
0
2
4
6
8
10
12
14
16
18
20
Eb/N0 (dB)
uncoded
coded (15,5)