EE 484

Homework 4

Due Monday, February 22, 2016

Work all 4 problems.

Problem 1. Pulse Coded Modulation (PCM) is to be used to encode a signal. The signal ranges between the values -4 and +4. There are 3 bits or 8 levels (hence 8 code numbers) available. The levels assigned have symmetry like we demonstrated in class. The first three sample values obtained (before quantization) are 2.3, 0.4, and -1.6, respectively.

a. Find the quantized values for the three sample values.

d. Find the corresponding PCM sequences for the quantized values.

Problem 2. Suppose we receive the analog signal

$$r_a(t) = A\cos(2\pi f t + \theta).$$

Here the amplitude A is a constant but we do *not* know its value. We do know that the frequency f is 100 Hz and the phase θ is $\pi/8$. We can follow the steps below to estimate the value of A. Assume for purposes of calculation that the value of A is 5, i.e., use $A = 5$ in the above signal in your calculations.

S1. Multiply $r_a(t)$ by $x(t)$, where $x(t) = \cos(2\pi 100t + \pi/8)$. Call the result $y(t)$.

S2. Integrate $y(t)$ from 0 to T and multiply the result by $2/T$. The result is your estimate of A.

a. Follow the 2 steps above and estimate A using $T = 6,\ 8,\ 16,\ 26,\ 126$ msec.

b. Explain why following the 2 steps above will give the exact answer for A as $T \to \infty$.

c. Determine (analytically) the *finite* values of T that will make your estimate for A exact and using the smallest such T follow the two steps above again to estimate A.

Problem 3. Suppose we receive the analog signal

$$r_a(t) = A\cos(2\pi 100t + \theta)$$

and sample it at 800 Hz to get the digital signal

$$r(n) = A\cos(0.25\pi n + \theta).$$

Here the amplitude A is a constant but we do *not* know its value. Furthermore, we do *not* know the θ phase value. We can follow the steps below to estimate the value of A. Assume for purposes of calculation that the value of A is 5 and $\theta = \pi/4$, i.e., use $A = 5$ and $\theta = \pi/4$ in the above signal in your calculations.

S1. Multiply $r(n)$ by $x_1(n)$ and $x_2(n)$, where $x_1(n) = \cos(0.25\pi n)$ and $x_2(n) = \sin(0.25\pi n)$. Call the results $y_1(n)$ and $y_2(n)$, respectively.

S2. Simply add up the values of $y_1(n)$ and $y_2(n)$ for $n = 0, 1, 2, \ldots N-1$ (some N) and take the average of each (divide by N) and then multiply the averages by 2. Call the results z_1 and z_2, respectively.

S3. Compute $\sqrt{z_1^2 + z_2^2}$. This is the estimate of A.

a. Follow the 3 steps above and estimate A using $N = 5, 11, 19$.

b. Explain why following the 3 steps above will give the exact answer for A as $N \to \infty$.

d. Determine (analytically) the *finite* values of N that will make your estimate for A exact and using the smallest such N follow the 3 steps above again to estimate A.

Problem 4. Suppose we receive the quantized digital signal

$$r_q(n) = \text{Round}\left[A\cos(0.25\pi n + \pi/8)\right]$$

where 'Round' means the samples are rounded to the nearest integer. The amplitude A is a constant but we do not know its value. Furthermore, we do not know that the phase is $\pi/8$. We can follow the steps below to estimate the value of A. [For purposes of calculation let A actually have the value 10.]

S1. Multiply $r_q(n)$ by $x_1(n)$ and $x_2(n)$, where $x_1(n) = \cos(0.25\pi n)$ and $x_2(n) = \sin(0.25\pi n)$. Call the results $y_1(n)$ and $y_2(n)$, respectively.

S2. Simply add up the values of $y_1(n)$ and $y_2(n)$ for $n = 0, 1, 2, \ldots N-1$ (some N) and take the average of each (divide by N) and then multiply the averages by 2. Call the results z_1 and z_2, respectively.

S3. Compute $\sqrt{z_1^2 + z_2^2}$. This is the estimate of A.

a. Follow the 3 steps above and estimate A using $N = 4$.

b. Repeat (a) for $N = 8$.

c. Based on your answers to parts (a) and (b) what would your estimate for A be if N is 4000. Explain why the estimate is not becoming exact even for very large N.