EE 484
Homework 3 Solutions

Problem 1. Compute the Fourier transform of the following

$$x(t) = \begin{cases} 1, & -1/2 \le t \le 1/2, \\ 0, & elsewhere \end{cases}$$

and sketch a plot of its magnitude in the frequency domain.

Solution:

$$X(f) = \int_{-1/2}^{1/2} e^{-j\pi ft} dt$$

which becomes

$$X(f) = \begin{cases} \dfrac{\sin \pi f}{\pi f}, & f \neq 0, \\ 1, & f = 0. \end{cases}$$

This is a sinc function centered at $f = 0$.

Problem 2. Suppose $f = 1000$ Hz. Compute the Fourier transform of the following

$$x(t) = \begin{cases} \cos(2\pi ft), & -1/2 \le t \le 1/2, \\ 0, & elsewhere \end{cases}$$

and sketch a plot of its magnitude in the frequency domain.

Solution: Using the result in Problem 1 and a Fourier transform property we can write immediately

$$X(f) = \begin{cases} \dfrac{\sin \pi(f - f_c)}{2\pi(f - f_c)} + \dfrac{\sin \pi(f + f_c)}{2\pi(f + f_c)}, & f \neq \pm f_c, \\ 1, & f = \pm f_c. \end{cases}$$

These are scaled sinc functions centered at $f = f_c$ and $f = -f_c$.

Problem 3. Suppose now using Matlab that you sample $x(t)$ in Problem 2 with a sampling rate of f_s Hz corresponding to a sample period of T_s sec where $T_s = 1/f_s$. Plot the DFT (discrete Fourier transform) using Matlab when

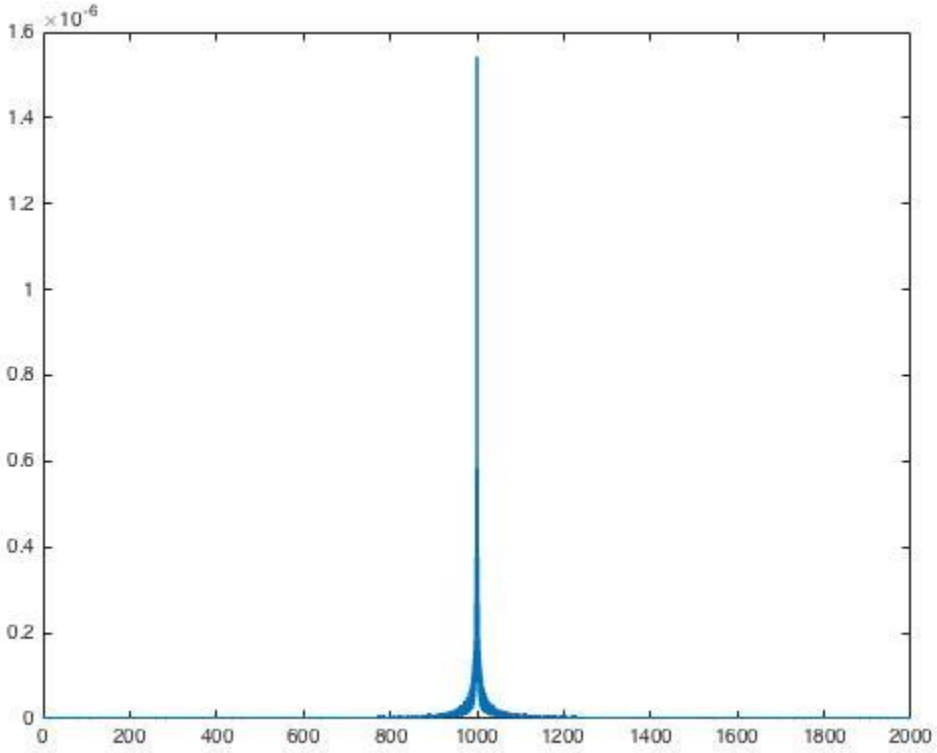
a. $f_s = 2000$ Hz.

b. $f_s = 2500$ Hz.

c. $f_s = 4000$ Hz.

d. $f_s = 4777$ Hz.

Solution: Plots are shown below.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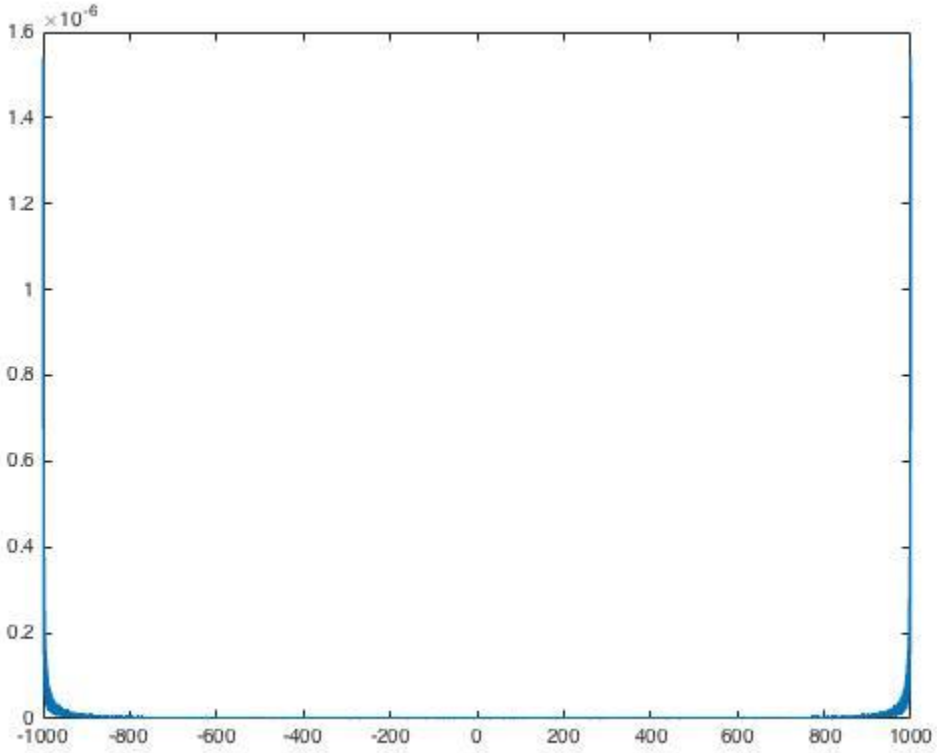
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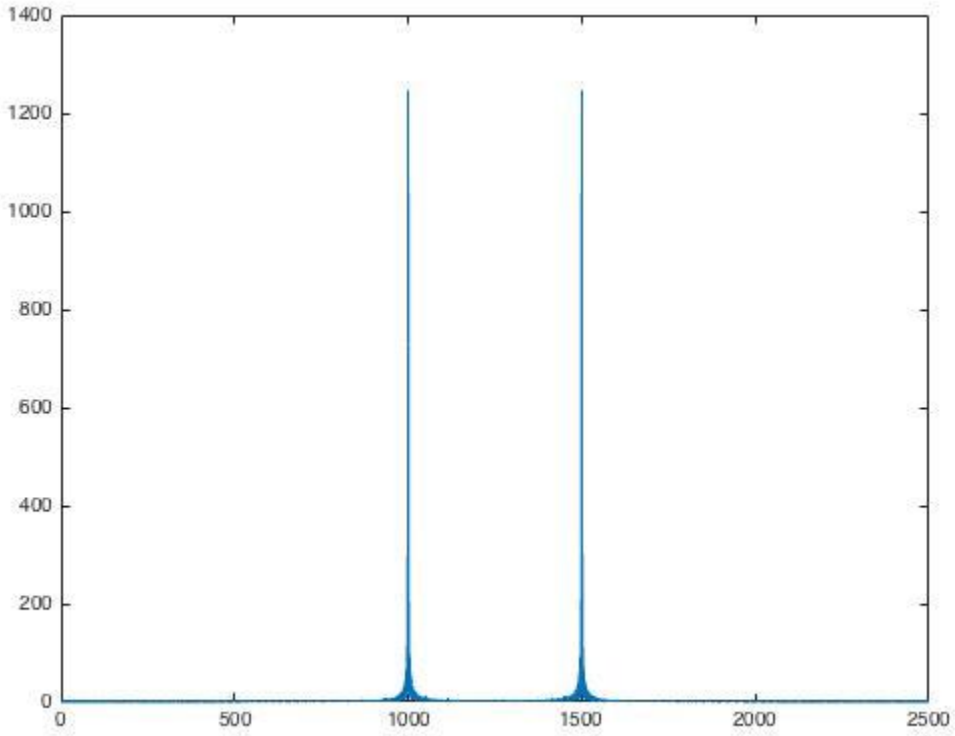
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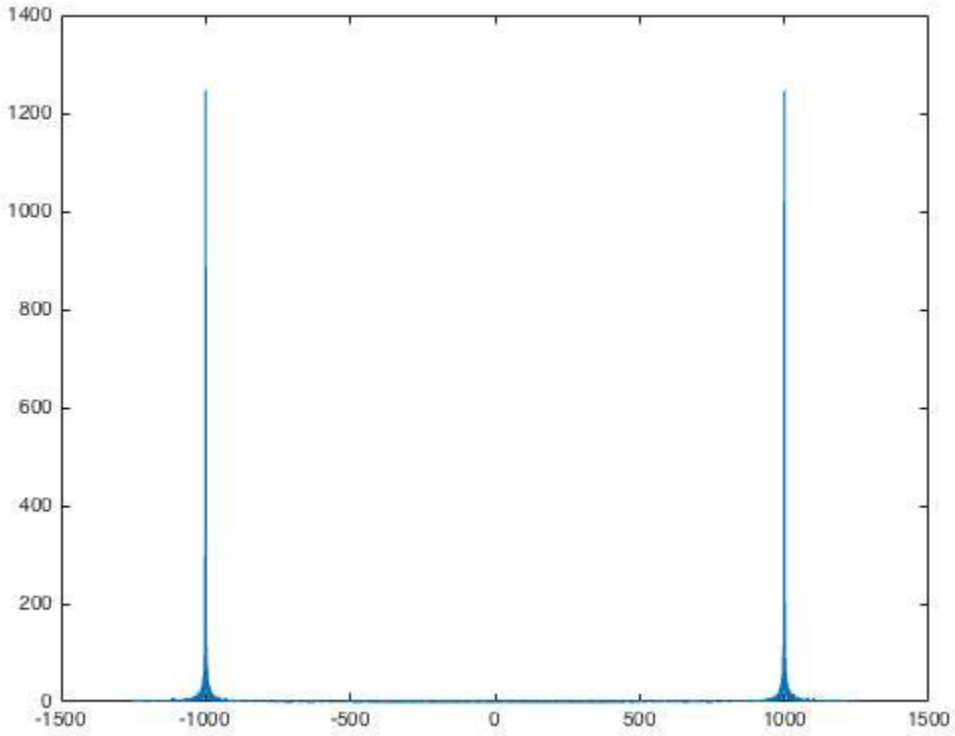
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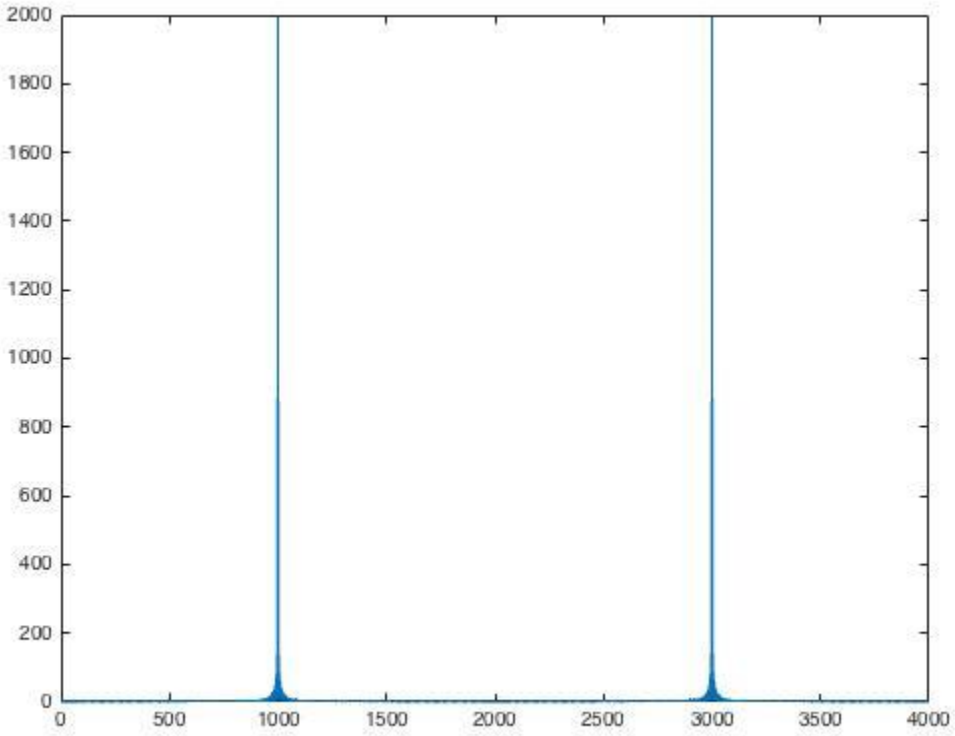
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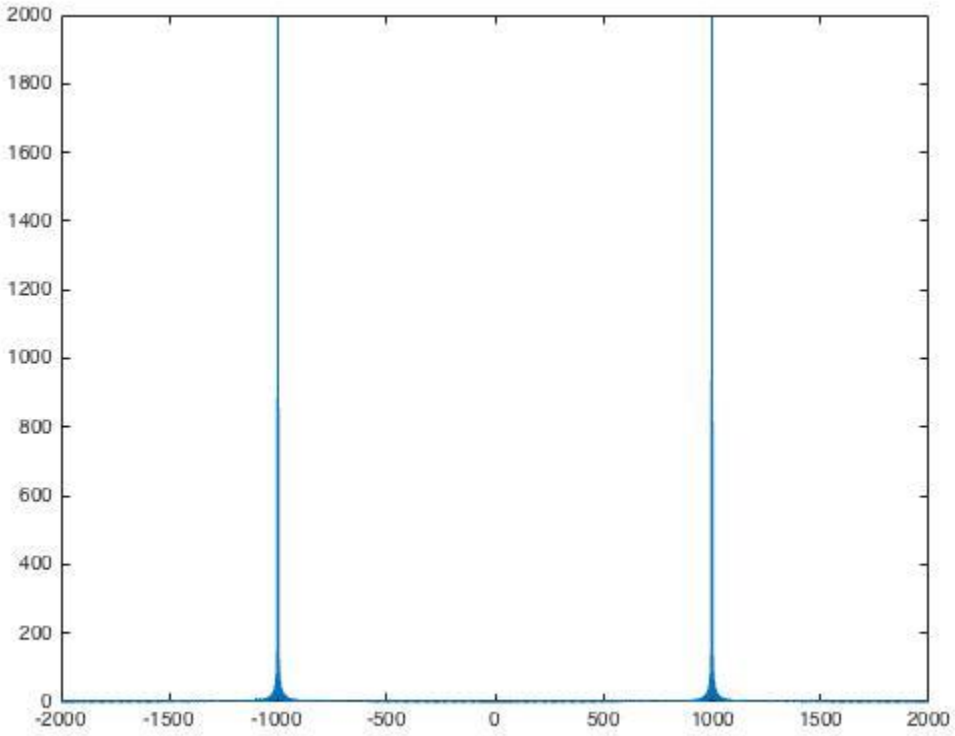
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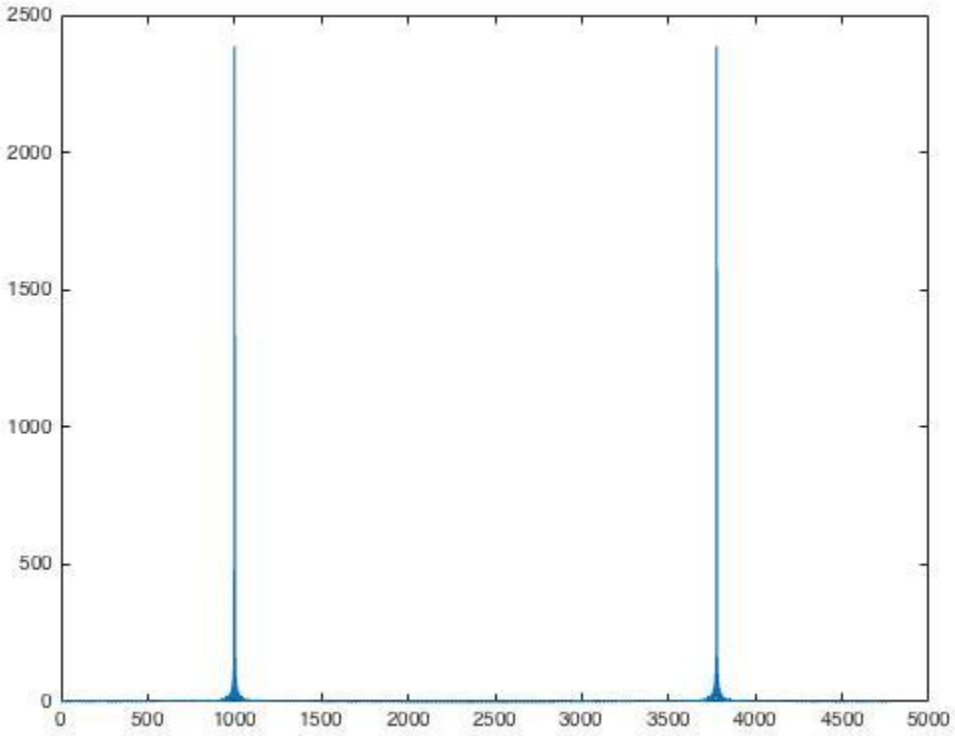
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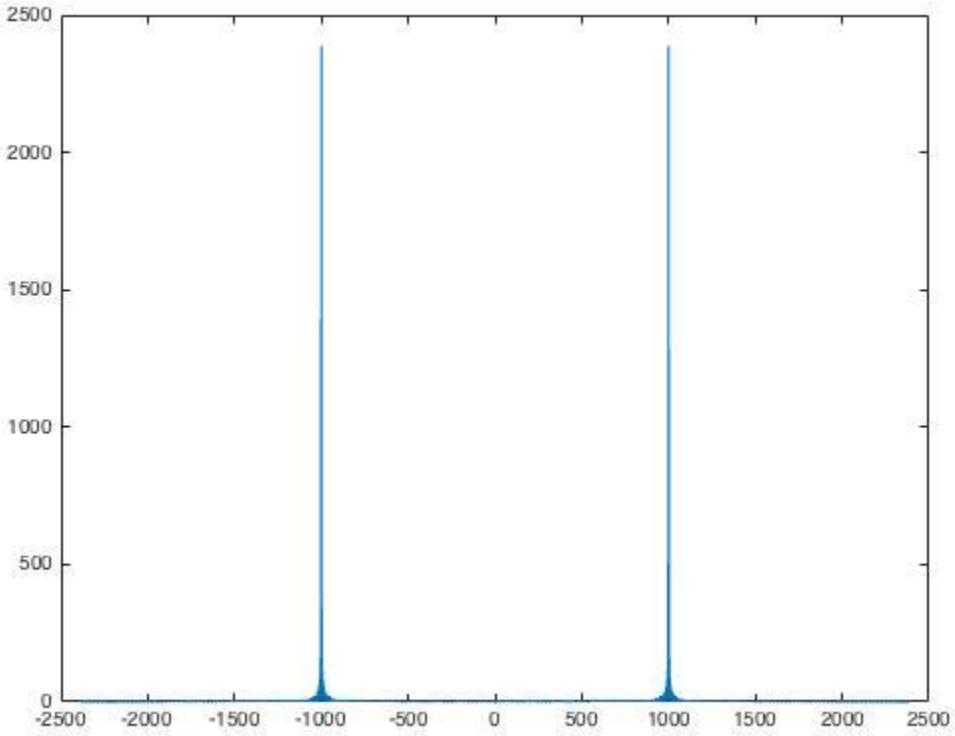
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