

# EE 484

## Homework 2

Due Monday, February 1, 2016

Work all 3 problems.

**Problem 1.** Let  $x(n)$  be an independent and identically distributed random sequence with each  $x(n)$  having mean 0 and variance  $\sigma^2$ . Suppose we form

$$y(n) = \sum_{k=0}^{\infty} h(k)x(n-k)$$

where,

$$h(n) = \begin{cases} (-\alpha)^{n/2}, & n \geq 0, \text{ even} \\ 0, & \text{elsewhere} \end{cases}$$

where  $0 < \alpha < 1$ . Find  $S_y(f)$ , the power spectral density of  $Y$ .

**Problem 2.** Consider a real Gaussian random sequence  $x(n)$ ,  $n$  an integer, with

$$E[x(n)] = 0, \quad E[x(n)^2] = 1, \quad E[x(n)x(m)] = \rho^{|n-m|}$$

where  $0 < \rho < 1$ . Let

$$y(n) = \begin{cases} \frac{x(n)}{n}, & n \neq 0 \\ 0, & n = 0. \end{cases}$$

- Is  $x(n)$  wide sense stationary?
- Find the covariance of  $y(n)$  and state whether or not it is wide sense stationary.

**Problem 3.** A zero mean sequence of i.i.d. random variables  $x(n)$  form the input to a causal linear system defined by the linear difference equation

$$y(n) = \alpha y(n-1) + x(n), \quad |\alpha| < 1.$$

Find the impulse response  $h(n)$  of a second system such that the sequence

$$z(n) = \sum_{i=0}^k h(i)y(n-i)$$

has a constant power spectral density, i.e.,

$$S_Z(f) = \sigma_z^2, \quad f \in \left[-\frac{1}{2}, \frac{1}{2}\right).$$