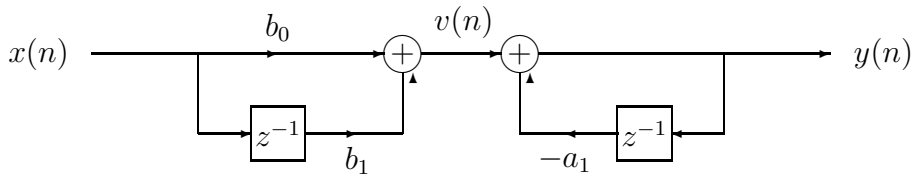
# 4.0 Implementation of Discrete-time Systems

We now discuss two realizations of the system

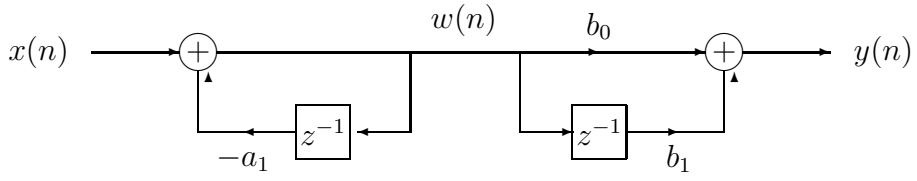
$$y(n) = -a_1 y(n-1) + b_0 x(n) + b_1 x(n-1).$$

This will describe by example the Direct Form I and Direct Form II realizations of discrete-time systems.




**Direct Form I Realization**

We first show that the above is equivalent to




We get

$$v(n) = b_0 x(n) + b_1 x(n-1) \text{ (non-recursive part)}$$
$$y(n) = -a_1 y(n-1) + v(n) \text{ (recursive part)}$$

$$w(n) = -a_1 w(n-1) + x(n)$$
$$y(n) = b_0 w(n) + b_1 w(n-1)$$

Let us now see that the 2nd implementation is equivalent to the first (can assume zero initial conditions).

1st implementation:

$$y(0) = v(0) = b_0 x(0)$$

2nd implementation:

$$y(0) = b_0 w(0) = b_0 x(0)$$

1st implementation:

$$y(1) = -a_1 y(0) + v(1) = -a_1 y(0) + b_0 x(1) + b_1 x(0)$$

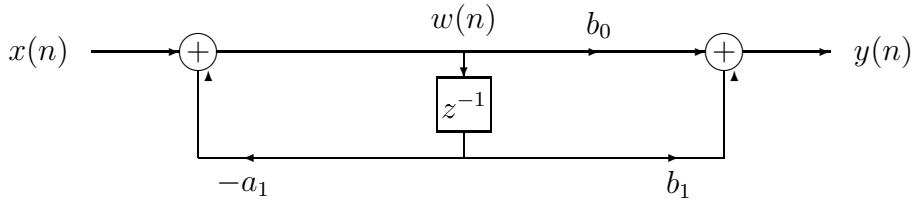
2nd implementation:

$$y(1) = b_0 w(1) + b_1 w(0) = b_0 \left(-a_1 w(0) + x(1)\right) + b_1 w(0)$$

$$= -a_1 y(0) + b_0 x(1) + b_1 x(0)$$

etc.

We thus get




**Direct Form II Realization**