2.0 A/D and D/A Converters

2.1 A/D Converters

In a digital computer all numbers are represented by a sequence of 0's and 1's. For example, using 3 bits a computer can represent $2^3 = 8$ different levels. If we used non-negative integers only we could process the numbers 0 thru 7 (000 to 111). If we allow negative numbers we could process -4 to +3 using 2's complement arithmetic. If we have 16 bits then $2^{16} = 65536$ levels are available. In this latter case we might want to do floating-point arithmetic so some of the bits would be reserved for the exponent of the number.

For illustration let

$$x_a(t) = A \sin 2\pi t, \quad 0 \leq t \leq 1.$$

Note that $F = 1$ Hz. Say we sample $x_a(t)$ every 1/8 seconds. Assume our A/D converter has only 8 levels with a spacing of $\Delta = A/3$.

We have

$$\text{Level } 3 \leftrightarrow A$$

so at $t = 1/8$ sec,

$$x_a(1/8) = A \sin 2\pi(1/8) = 0.707A$$

and $0.707(3) = 2.12$ so

"Level" $2.12 =$ Level $2 \leftrightarrow x_a(1/8) \Rightarrow x_a(1/8)$ is quantized to Level 2.

Actual A/D converters may either quantize to the nearest level (round) or truncate to the next lowest level. We are assuming in this example a rounding operation.

Level	2's Complement
3	011
2	010
1	001
0	000
-1	111
-2	110
-3	101
-4	100

Quantized Samples	Computer Representation
$x(0) = 0$	000
$x(1) = 2$	010
$x(2) = 3$	011
$x(3) = 2$	010
$x(4) = 0$	000
$x(5) = -2$	110
$x(6) = -3$	101
$x(7) = -2$	110
$x(8) = 0$	000

When we process this data we know that a 3 (011) is really an A and each sample is spaced 1/8 of a second apart. Usually we have many levels available so that the quantization error is small.

Problem. What if the amplitude A was suddenly increased by 25% but we did not change the A/D settings? Now

$$x_a(t) = \frac{5}{4} A \sin 2\pi t.$$

Here, any samples above A will be quantized to A. We will have *clipping* of the signal.

In practice, the levels of the A/D converter can be set to a fixed spacing and a gain stage is used to avoid clipping (at least most of the time) for the expected signal characteristics (in reality, the signal is "noisy" so the peak value can only be characterized statistically).

Note that choosing the correct gain setting requires careful considerations to avoid loss of resolution of the signal.

This process of setting up the A/D converter (gain and spacing levels) is called "loading the A/D." More sophisicated systems employ algorithms to analyze the incoming signal characteristics and automatically adjust the gain setting (up or down) appropriately.

Typical A/D converters use 8 to 12 bits (so 256 to 4096 levels are available).

2's Complement Arithmetic

n	2's Complement
3	011
2	010
1	001
0	000
-1	111
-2	110
-3	101
-4	100

To compute $3 - 1$ we take $(011) + (111) = 1010$ and we ignore the leading 1 and take the rightmost 3 bits to get 010 $(= 2)$.

To find the 2's complement of $-k$, take the binary representation of $+k$, complement each bit, i.e., take the 1's complement, and add 1. For example, to find the 2's complement of -2 we take 010 $(= +2)$ and complement each bit to get 101, add 1 to get 110 which is the 2's complement representation for -2.

2.2 D/A Converters

The technique to be discussed in class is called *sample and hold*. One can use other techniques as well such as linear interpolation.