Name: _____ **Student Number:** _____

EE 113 Midterm
Spring 2007

Inst: Dr. C.W. Walker

Problem	Points	Score
1	9	
2	11	
3	15	
4	15	
5	10	
6	10	
7	10	
8	10	
9	10	
Total	100	

Instructions and Information:

1) Print your name and student number at the top of the page.

2) Make sure your exam has 9 problems and 17 numbered pages. Do not write any work to be graded on the back of the pages.

3) This is a closed book exam. You may use one sheet of notes (front and back). You may also use a calculator.

4) Partial credit will be given but you must **show your work where appropriate or justify your answers to receive any credit.**

5) **Circle or box your final answers.**

Problem 1. For parts (a), (b) and (c) determine whether or not the system is

i. linear

ii. time-invariant

iii. BIBO stable, i.e., bounded input-bounded output stable

a. $y(n) = 2x(n)$.

b. $y(n) = \log(n+1)u(n)$, log is the natural logarithm.

c. $y(n) = [x(n)]^n$.

Note: You do not need to show any work on this problem if you can quickly recognize the answer.

Problem 1 (extra workspace).

Problem 2. Consider the system described by the following difference equation:

$$y(n) - \frac{7}{12}y(n-1) + \frac{1}{12}y(n-2) = x(n),$$

where,

$$x(n) = (1/2)^n\, u(n), \quad y(-1) = 1,\ y(-2) = 0.$$

a. Find the homogeneous solution for this system.

b. Find the particular solution for this system.

c. Find the complete solution for this system.

d. Evaluate your $y(n)$ for $n = 0, 1, 2$.

Problem 2 (extra workspace).

Problem 3. Compute $X(z)$, the forward z-transform, (if it exists) for each of the following. Remember to specify the region of convergence in each case. If the forward z-transform does not exist, explain why.

a. $x(n) = \left(\frac{1}{2}\right)^n u(n-2)$.

b. $x(n) = nu(n)$.

c.

$$x_1(n) = \begin{cases} \alpha^n u(n), & n \text{ is a multiple of 2,} \\ 0, & \text{elsewhere.} \end{cases}$$

$x(n) = x_1(2n)$.

Problem 3 (extra workspace).

Problem 4. For parts (a) and (b) of the following compute $x(n)$, the inverse z-transform, using any method you wish. For part (c) use the residue method.

a. $X(z) = \dfrac{z^4 - 1}{z - \frac{1}{4}}$, ROC corresponds to a right-sided sequence.

b. $X(z) = \dfrac{z}{z^2 - 7z + 12}$, $\text{ROC} = \{z : |z| > 4\}$.

c. $X(z) = \dfrac{z}{(z - \frac{1}{2})(z - \frac{1}{4})}$, $\text{ROC} = \left\{z : \dfrac{1}{4} < |z| < \dfrac{1}{2}\right\}$.

Evaluate your expression for $x(n)$ at $n = 0, 1, 2$ in each case.

Problem 4 (extra workspace).

Problem 5. Evaluate the following infinite sum:

$$S = \sum_{n=0}^{\infty} n \left(\frac{1}{3}\right)^n .$$

Problem 6. Suppose $x(n)$ is a real ($x(n) = x^*(n)$) and even ($x(n) = x(-n)$) sequence with z-transform $X(z)$. Suppose z_0 is a zero of $X(z)$, i.e., $X(z_0) = 0$ for some complex number z_0.

a. Show $1/z_0$ is also a zero of $X(z)$.

b. Are there any other zeros of $X(z)$ implied by the information given? If so, find them.

Problem 7. Suppose $H(z)$, the z-transform of $h(n)$, is

$$H(z) = \frac{z}{z - \frac{1}{4}}, \quad |z| > \frac{1}{4}$$

and you are given the sequence

$$h_2(n) = \begin{cases} nh(n), & n \text{ even}, \\ 0, & n \text{ odd}. \end{cases}$$

Find $H_2(z)$.

Problem 8. A proposed form for a z-transform of a signal, $x(n)$, is given as

$$X(z) = \frac{z}{z-\alpha} + \frac{z}{z-\beta}$$

subject to the following constraints:

1. $|\alpha| \neq |\beta|$.
2. $\alpha + \beta = 3$.
3. $x(n)x(-n) = -2$ when $n = 1$.

Given the constraints, if their are any valid regions of convergence for $X(z)$, find them. If none exist, then show why not. You may assume without loss of generality that $|\alpha| > |\beta|$.

Problem 8 (extra workspace).

Problem 9.

a. Consider the discrete-time signal

$$x(n) = n^{\log n} u(n-1)$$

where log denotes the natural logarithm. Determine if the z-transform of this signal exists and if it does exist find the region of convergence. Note you do not have to actually find the z-transform. Justify your answer for credit.

b. Consider the discrete-time signal

$$x_N(n) = n^{\log n} u(n-N).$$

Find the function $X_N(z)$ that the z-transform of this signal approaches as N approaches infinity.

Problem 9 (extra workspace).

Extra workspace. If you use this space for work to be graded reference it from the given problem.

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