Name: ________________________ **Student Number:** ____________________

EE 113 Final

Spring 2007

Inst: Dr. C.W. Walker

Problem	Points	Score	Problem	Points	Score
1	10		6	10	
2	10		7	10	
3	10		8	10	
4	10		9	10	
5	10		10	10	
Total				100	

Instructions and Information:

1) Print your name and student number at the top of the page.

2) Make sure your exam has 10 problems.

3) This is a closed book exam. You may use two sheets of notes (front and back). You may also use a calculator.

4) Partial credit will be given but you must **show your work where appropriate or justify your answers to receive any credit.**

5) **Circle or box your final answers.**

Problem 1. Consider the system described by the following difference equation:

$$y(n) - \frac{5}{6}y(n-1) + \frac{1}{6}y(n-2) = x(n),$$

where,

$$x(n) = 2u(n), \quad y(-1) = 1, \ y(-2) = 0.$$

a. Find a closed form expression for $y(n)$ using the unilateral z-transform.

b. Evaluate your $y(n)$ for $n = 0, 1, 2$.

Problem 1 (extra workspace).

Problem 2. Suppose $H(z)$, the z-transform of $h(n)$, is

$$H(z) = \frac{z}{z - \frac{1}{2}}, \quad |z| > \frac{1}{2}$$

and you are given the sequence

$$h_2(n) = \begin{cases} h(n), & n \text{ even}, \\ 0, & n \text{ odd}. \end{cases}$$

Find $H_2(z)$.

Problem 3. Consider

$$y(n) = \begin{cases} x\left(\frac{n}{3}\right), & n \text{ a multiple of } 3 \\ 0, & \text{elsewhere.} \end{cases}$$

Here $y(n)$ is taken to be a length $3N$ sequence and $x(n)$ is of length N.

Find $Y(k)$, the DFT of $y(n)$, in terms of $X(k)$, the DFT of $x(n)$.

Problem 4. A certain LTI system of the form

$$H(z) = \frac{(z - z_0)(z - z_1)}{(z - p_0)(z - p_1)}$$

with two zeros and two poles is known to have the unit circle inside its region of convergence. The zeros are located at $z = 0$ (the origin) and $z = 1/2$. The locations of the poles are unknown but it is known that the poles form a complex conjugate pair.

A test is performed and it is determined that the frequency response is 3/4 at both $\omega = 0$ and $\omega = \pi$.

a. Find the locations of the two poles in the z-plane.

b. Find the difference equation that corresponds to this system. If you are unable to work part (a) then let $p_0 = \frac{1}{5} + \frac{\sqrt{3}}{5}i$ (not the right answer) to work this part.

Problem 5. Compute $X(z)$, the forward z-transform, (if it exists) for each of the following. Simplify your answer as much as possible. Remember to specify the region of convergence in each case. If the forward z-transform does not exist, explain why.

a. $x(n) = \left(\frac{1}{2}\right)^n u(n-3)$.

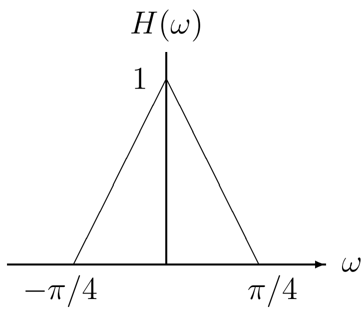
b. $x(n) = n^2 u(n-1)$.

c.
$$x(n) = \begin{cases} \alpha^n u(n), & n \text{ is a multiple of } k, \\ \beta^n u(n), & \text{elsewhere,} \end{cases}$$

where, k is a positive integer.

Problem 5 (extra workspace).

Problem 6. The DTFT for a certain filter is shown below. Find $h(n)$.



Problem 7. Suppose an output $y(n)$ is formed as

$$y(n) = (1 - \beta) \cdot z(n) + \beta y(n-1),$$

where

$$z(n) = (1 - \alpha) \cdot Q[x(n)] + \alpha z(n-1),$$

where, α and β are positive and real, $0 < \alpha < 1$, $0 < \beta < 1$, and $Q[x(n)]$ represents a quantized $x(n)$ produced by an A/D converter using b fractional bits (you may assume $|x(n)| < 1$). If $\alpha = 0.5$, $\beta = 0.9$ and $b = 4$ bits, find the output noise variance due to quantization noise.

Problem 7 (extra workspace).

Problem 8. A Butterworth filter is to have a 3-dB cutoff frequency of 4 KHz and an attenuation of at least 15 dB at three times the cutoff frequency. Find n, the order of the filter.

Problem 9. A Butterworth filter is to have a 3-dB cutoff frequency of 4 KHz and an attenuation of at least 20 dB at four times the cutoff frequency.

a. Find $H_n(s)$, the normalized transfer function for this Butterworth filter.

b. Find $H(s)$, the unnormalized transfer function for this Butterworth filter.

Some Butterworth polynomial table results:

$$n = 2,\ a_1 = 1.4142$$

$$n = 3,\ a_1 = 2.0, \quad a_2 = 2.0$$

$$n = 4,\ a_1 = 2.6131, \quad a_2 = 3.4142, \quad a_3 = 2.6131$$

Problem 9 (extra workspace).

Problem 10. Suppose we have the discrete-time signal

$$r(n) = A\cos(2\pi f n + \theta).$$

Here the amplitude A is a constant but we do *not* know its value. Also, we do *not* know what the frequency is and we do *not* know the phase θ.

Suppose someone takes the discrete Fourier transform of this signal and by looking at the spectrum estimates the frequency f to be 47 Hz. However, suppose the actual frequency is 50 Hz. Using the 47 Hz estimate for f describe and implement (using equations) an algorithm for estimating the amplitude A. Explain analytically the effect of using the wrong frequency in your algorithm (i.e., how does using the wrong frequency affect your estimate of A). If you use integration in your algorithm does the frequency error affect how long you might integrate? If so, explain why?

Problem 10 (extra workspace).

Extra workspace. If you use this space for work to be graded reference it from the given problem.

Extra workspace. If you use this space for work to be graded reference it from the given problem.

Extra workspace. If you use this space for work to be graded reference it from the given problem.