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EE 132A

Homework 4 Solutions

Problem 1. Proakis & Salehi 6.3 except replace $S_m(f)$ in Figure P-6.3 with

$$S_m(f) = P_0 \Lambda\left(\frac{f - f_c}{W}\right) + P_0 \Lambda\left(\frac{f + f_c}{W}\right)$$

Solution: One way of solving this problem is to note that the output SNR for DSB-SC is equal to the baseband SNR. Then we have

$$\left(\frac{S}{N}\right)_o = \left(\frac{S}{N}\right)_b = \frac{P_R}{N_0 W}$$

where P_R is the recieved power, in this case

$$P_R = \int_{-\infty}^{\infty} S_m(f) df = 2WP_0$$

So finally

$$\left(\frac{S}{N}\right)_o = \frac{2WP_0}{N_0 W} = \frac{2P_0}{N_0}$$

Another way of solving the problem is noting that for DSB-SC, we have

$$\left(\frac{S}{N}\right)_o = \frac{A_c^2 P_m}{2N_0 W}$$

But $A_c^2 P_m/2$ is the received signal power, or $P_R = 2WP_0$ and therefore

$$\left(\frac{S}{N}\right)_o = \frac{2P_0}{N_0}$$

Problem 2. Proakis & Salehi 6.5 except change the modulation index from 0.5 to 0.7 and in Figure P-6.5 (b) change 1500 to 1250.

Solution: 1) Since $|H(f)| = 1$ for $f = f_c \pm f_m$, the signal at the output of the noise-limiting filter is

$$r(t) = 10^{-3}[1 + \alpha \cos(2\pi f_m t + \phi)] \cos(2\pi f_c t) + n(t)$$

The signal power is

$$\begin{aligned} P_R &= \lim_{T\to\infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} 10^{-6}[1 + \alpha \cos(2\pi f_m t + \phi)]^2 \cos^2(2\pi f_c t) dt \\ &= \frac{10^{-6}}{2}[1 + \frac{\alpha^2}{2}] = 6.225 \times 10^{-7} \end{aligned}$$

The noise power at the output of the noise-limiting filter is

$$P_{n,o} = \frac{N_0}{2}\int_{-\infty}^{\infty} |H(f)|^2 df = \frac{N_0}{2} \times 2 \times (2000 + 2 \times \frac{250}{3}) = 4.5 \times 10^{-9}$$

where we have used the fact that $\int_0^{250}(1 - f/250)^2 df = 250/3$

2)The multiplication of $r(t)$ by $2\cos(2\pi f_c t)$ yields

$$y(t) = \frac{10^{-3}}{2}[1 + \alpha\cos(2\pi f_m t + \phi)]2 + \frac{1}{2}n_c(t)2 + \text{double frequency terms}$$

The LPF rejects the double frequency components and therefore, the output of the filter is

$$v(t) = 10^{-3}[1 + \alpha\cos(2\pi f_m t)] + n_c(t)$$

If the DC component is blocked, then the signal power at the output of the LPF is

$$P_o = \frac{10^{-6}}{2}0.7^2 = 2.45 \times 10^{-7}$$

whereas, the output noise power is

$$P_{n,o} = P_{n_c} = P_n = 2\frac{N_0}{2}2000 = 4 \times 10^{-9}$$

where we have used the fact that the lowpass filter has a bandwidth of 1000 Hz. Hence, the output SNR is

$$SNR = 61.25 = 17.9 \text{ dB}$$

Problem 3. Proakis & Salehi 6.8 except in part (4) change the modulation index to 0.7 and change the normalized message power to 0.25.

Solution: 1) The received power is

$$P_R = \frac{P_T}{10^{80/10}} = \frac{4 \times 10^4}{10^8} = 4 \times 10^{-4} \text{ W}$$

If the transmitted signal has a bandwidth B_c, the predetection signal-to-noise ratio is

$$\left(\frac{S}{N}\right) = \frac{P_R}{N_0 B_c}$$

In the case of DSB-SC and conventional AM, we have $B_c = 2W$ and therefore

$$\left(\frac{S}{N}\right)_{\text{DSB-SC,AM}} = \frac{P_R}{2N_0 W} = 100$$

In the case of SSB, we have $B_c = W$ and therefore

$$\left(\frac{S}{N}\right)_{\text{SSB}} = 200$$

2) For DSB-SC, the output SNR is

$$\left(\frac{S}{N}\right)_o = \left(\frac{S}{N}\right)_b = \frac{P_R}{N_0 W} = 200$$

3) For SSB, the output SNR is

$$\left(\frac{S}{N}\right)_o = \left(\frac{S}{N}\right)_b = \frac{P_R}{N_0 W} = 200$$

4) For AM with modulation index $\alpha = 0.7$ and normalized message power $P_{m_n} = 0.25$, the output SNR is

$$\left(\frac{S}{N}\right)_o = \left(\frac{S}{N}\right)_b \frac{\alpha^2 P_{m_n}}{1+\alpha^2 P_{m_n}} = \frac{P_R}{N_0 W}\frac{\alpha^2 P_{m_n}}{1+\alpha^2 P_{m_n}} = 21.83$$

Problem 4. Suppose an AM communication signal has $R_m(t) = 8\text{sinc}^2(5000t)$ and $|m(t)| = 4$. The channel the signal is transmitted over has 60 dB attenuation and the noise has spectral density $S_n(f) = \frac{N_0}{2} = 10^{-10}$ W/Hz. We want the output of the demodulator at the receiver to be 45 dB. Find the required transmit power and bandwidth for the following cases.

a. DSB-SC AM.

b. SSB AM.

Solution:

a) The power spectral density of the message is

$$S_m(f) = \mathcal{F}[R_m(t)] = \frac{8}{5000}\Lambda(f/5000)$$

Therefore the bandwidth of the message is $W = 5000$. For DSB-SC, the output SNR is

$$\left(\frac{S}{N}\right)_o = \left(\frac{S}{N}\right)_b = \frac{P_R}{N_0 W} = 10^{45/10}$$

Thus the received signal power must be at least $P_R = 0.0316$ to ensure an SNR of 45 dB.

The transmitted signal power must be at least

$$P_T = P_R \times 10^6 = 3.16 \times 10^4 = 31.6 \text{ kW}$$

The bandwidth of the transmitted signal is $B_c = 2W = 10$ kHz.

b) For SSB, again we have

$$\left(\frac{S}{N}\right)_o = \left(\frac{S}{N}\right)_b = \frac{P_R}{N_0 W} = 10^{45/10}$$

and therefore $P_R = 0.0316$. The transmitted signal power must be at least

$$P_T = P_R \times 10^6 = 3.16 \times 10^4 = 31.6 \text{ kW}$$

The bandwidth of the transmitted signal is $B_c = W = 5$ kHz.