EE 132A

Homework 1

Due Wednesday January 23, 2008

Work all 14 problems.

Problem 1. For $a \neq 0$ prove the scaling property of the Fourier transform:

$$x(at) \longleftrightarrow \frac{1}{|a|} X(f/a).$$

Problem 2. Prove the time-shift property of the Fourier transform:

$$x(t - t_0) \longleftrightarrow e^{-2\pi f t_0} X(f).$$

Problem 3.

a. Compute the Fourier transoform of the rectangular pulse, $x(t) = \prod(t)$, and sketch a plot of its graph in the frequency domain. You should produce two plots (one for the magnitude of the Fourier transform and one for the phase). In the magnitude plot, clearly identify where the maximum height occurs and the value of the maximum height. Also, identify where the zero-crossings occur.

b. Repeat part a for $x(t) = \prod(t) \cos(2\pi f_0 t)$ where $f_0 = 1$ kHz.

Problem 4. Proakis & Salehi 2.1 parts 1, 5, 7.

Problem 5. Proakis & Salehi 2.39 part 5.

Problem 6. Using properties of the Fourier transform compute

$$\int_{-\infty}^{\infty} \left(\frac{\sin \pi f}{\pi f} \right)^2 df.$$

Problem 7. Let $\alpha > 0$, and consider the convolution

$$x(t) = e^{\alpha t}u(-t) * e^{-\alpha t}u(t)$$

where $u(t)$ is the unit-step function.

a. Compute $x(t)$ via direct convolution.

b. Compute $x(t)$ using a Fourier transform property.

Problem 8. Proakis & Salehi 2.52.

Problem 9. Proakis & Salehi 2.53.

Problem 10. Proakis & Salehi 2.7.

Problem 11. Proakis & Salehi 2.9.

Problem 12. Proakis & Salehi 2.8 parts 2, 3, 4.

Problem 13. Compute the Fourier transform of

$$f(t) = \sum_{n=-\infty}^{\infty} k_n \delta(t-n)$$

where

a. $k_n = (-1)^n$.

b. $k_n = p(1-p)^n$ for $n \geq 0$ and is 0 elsewhere, $(0 < p < 1)$.

Problem 14. Compute the Hilbert transform of

a. $f_1(t) = \cos t$.

b. $f_2(t) = \sin t$.

c. $f_3(t) = \prod(t)$.

d. $f_4(t) = \delta(t)$.

e. $f_5(t) = \dfrac{\sin t}{t}$.

f. $f_6(t) = \dfrac{1}{1+t^2}$.