# EE 567
## Handout 3

Let $n(t)$ be a WSS stochastic process with zero mean. Then

$$\begin{aligned} n(t) &= a(t)\cos[2\pi f_c t + \theta(t)] \\ &= x(t)\cos 2\pi f_c t - y(t)\sin 2\pi f_c t \\ &= Re[z(t)e^{i2\pi f_c t}] \end{aligned}$$

$$a(t) = envelope$$

$$z(t) = x(t) + iy(t) \text{ (complex envelope)}$$

$$E[n(t)] = 0 \Longrightarrow E[x(t)] = E[y(t)] = 0$$

Claim

$$R_X(\tau) = R_Y(\tau)$$

$$R_{XY}(\tau) = -R_{YX}(\tau)$$

<u>Proof</u>

$$R_n(\tau) = E[n(t)n(t-\tau)]$$

$$= E[(x(t)\cos 2\pi f_c t - y(t)\sin 2\pi f_c t)(x(t-\tau)\cos 2\pi f_c(t-\tau) - y(t-\tau)\sin 2\pi f_c(t-\tau))]$$

$$= R_X(\tau)\cos 2\pi f_c t \cos 2\pi f_c(t-\tau)$$

$$+R_Y(\tau)\sin 2\pi f_c t \sin 2\pi f_c(t-\tau)$$

$$-R_{YX}(\tau)\sin 2\pi f_c t \cos 2\pi f_c(t-\tau)$$

$$-R_{XY}(\tau)\cos 2\pi f_c t \sin 2\pi f_c(t-\tau)$$

Use

$$\cos A \cos B = \frac{1}{2}[\cos(A-B) + \cos(A+B)]$$

$$\sin A \sin B = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$$

$$\sin A \cos B = \frac{1}{2}[\sin(A-B) + \sin(A+B)]$$

Then

$$
\begin{aligned}
R_n(\tau) &= \frac{1}{2}[R_X(\tau) + R_Y(\tau)] \cos 2\pi f_c \tau \\
&+ \frac{1}{2}[R_X(\tau) - R_Y(\tau)] \cos 2\pi f_c (2t - \tau) \\
&- \frac{1}{2}[R_{YX}(\tau) + R_{XY}(\tau)] \sin 2\pi f_c \tau \\
&- \frac{1}{2}[R_{YX}(\tau) + R_{XY}(\tau)] \sin 2\pi f_c (2t - \tau)
\end{aligned}
$$

RHS must be independent of $t$ for $n(t)$ to be WSS. Thus,

$$R_X(\tau) = R_Y(\tau)$$

and

$$R_{XY}(\tau) = -R_{YX}(\tau)$$