EE 567

Homework 7

Due Tuesday, October 17, 2018 at 6:40 p.m.

Work all 5 problems.

Problem 1. A signal of the form

$$s_1(t) = A\cos(2\pi f_c t + \theta)$$

is transmitted to a receiver. The signal waveform is T seconds long. A multipath version of the signal, $s_2(t)$, delayed by τ seconds also arrives at the receiver (you can think of this multipath signal as being just like $s_1(t)$ except it is delayed and the amplitude is possibly scaled). The cross-correlation of the two real signals is defined by

$$\gamma_{12} = \frac{1}{T}\int_0^T s_1(t)s_2(t)dt.$$

Determine the smallest value of τ that can be tolerated to ensure that the cross-correlation of the direct path and multipath signal is less than 5% of the direct signal power. Assume that $f_c T \gg 1$ and the multipath signal has an amplitude that is 50% that of the direct signal when received.

Problem 2. In class we had that the gain for a parabolic antenna is

$$g = \left(\frac{\pi}{3}\right)^2 \left(\frac{df_c}{10^8}\right)^2$$

and the half-power beamwidth is

$$\phi_b = 3.06\left(\frac{10^8}{df_c}\right) \quad \text{radians.}$$

The actual gain of the antenna will be reduced if there is any pointing error ϕ_e and the resulting equation is

$$g(\phi_e) = g\left[\frac{2J_1(\pi d\phi_e/\lambda)}{(\pi d\phi_e/\lambda)}\right]^2$$

where λ is the wavelength. Using the Bessel function approximation for small argument

$$J_1(x) \approx \frac{x}{2}\left(1 - \frac{x^2}{8}\right) \approx \frac{x}{2}e^{-x^2/8}$$

show that

$$g(\phi_e) \approx g e^{-2.6(\phi_e/\phi_b)^2}.$$

Plot the antenna gain (in dB) versus d/λ for $\phi_e = 0,\ 0.05,\ 0.1,\ 0.2,\ 0.3$ degrees. You should plot each of these on the same graph. Vary d/λ from 10 to 1000. You should be able to deduce from your results that pointing errors prevent the use of very narrow beams.

Problem 3. Suppose we receive the analog signal

$$r_a(t) = A\cos(2\pi f_c t + \theta).$$

Here the amplitude A is a constant but we do *not* know its value. We do know that the frequency f_c is 200 Hz and the phase θ is $\pi/4$. We can follow the steps below to estimate the value of A. Assume for purposes of calculation that the value of A is 5, i.e., use $A = 5$ in the above signal in your calculations.

S1. Multiply $r_a(t)$ by $x(t)$, where $x(t) = \cos(2\pi f_c t + \pi/4)$. Call the result $y(t)$.

S2. Integrate $y(t)$ from 0 to T and multiply the result by $2/T$. The result is your estimate of A.

a. Follow the 2 steps above and estimate A using $T = 6,\ 8,\ 16,\ 26,\ 126$ msec.

b. Explain why following the 2 steps above will give the exact answer for A as $T \to \infty$.

c. Determine (analytically) the *finite* values of T that will make your estimate for A exact and using the smallest such T follow the two steps above again to estimate A.

Problem 4. Suppose we receive the analog signal

$$r_a(t) = A\cos(2\pi 200t + \theta)$$

and sample it at 1600 Hz to get the digital signal

$$r(n) = A\cos(0.25\pi n + \theta).$$

Here the amplitude A is a constant but we do *not* know its value. Furthermore, we do *not* know the θ phase value. We can follow the steps below to estimate the value of A. Assume for purposes of calculation that the value of A is 5 and $\theta = \pi/4$, i.e., use $A = 5$ and $\theta = \pi/4$ in the above signal in your calculations.

S1. Multiply $r(n)$ by $x_1(n)$ and $x_2(n)$, where $x_1(n) = \cos(0.25\pi n)$ and $x_2(n) = \sin(0.25\pi n)$. Call the results $y_1(n)$ and $y_2(n)$, respectively.

S2. Simply add up the values of $y_1(n)$ and $y_2(n)$ for $n = 0, 1, 2, \ldots N-1$ (some N) and take the average of each (divide by N) and then multiply the averages by 2. Call the results z_1 and z_2, respectively.

S3. Compute $\sqrt{z_1^2 + z_2^2}$. This is the estimate of A.

a. Follow the 3 steps above and estimate A using $N = 5, 11, 19$.

b. Explain why following the 3 steps above will give the exact answer for A as $N \to \infty$.

d. Determine (analytically) the *finite* values of N that will make your estimate for A exact and using the smallest such N follow the 3 steps above again to estimate A.

Problem 5. Use Matlab. Suppose we receive the analog signal

$$r_a(t) = A\cos(2\pi 200t + \theta)$$

and sample it at 1600 Hz to get the digital signal

$$r(n) = A\cos(0.25\pi n + \theta).$$

Suppose now we quantize the digital signal to get

$$r_q(n) = \text{Round}\left[A\cos(0.25\pi n + \theta)\right]$$

where 'Round' means the samples are rounded to the nearest integer. The amplitude A is a constant but we do not know its value. Furthermore, we do not know that the phase is $\theta = \pi/4$. We can follow the steps below to estimate the value of A. [For purposes of calculation let A actually have the value 10.]

S1. Multiply $r_q(n)$ by $x_1(n)$ and $x_2(n)$, where $x_1(n) = \cos(0.25\pi n)$ and $x_2(n) = \sin(0.25\pi n)$. Call the results $y_1(n)$ and $y_2(n)$, respectively.

S2. Simply add up the values of $y_1(n)$ and $y_2(n)$ for $n = 0, 1, 2, \ldots N - 1$ (some N) and take the average of each (divide by N) and then multiply the averages by 2. Call the results z_1 and z_2, respectively.

S3. Compute $\sqrt{z_1^2 + z_2^2}$. This is the estimate of A.

a. Follow the 3 steps above and estimate A using $N = 4$.

b. Repeat (a) for $N = 8$.

c. What values of N makes your estimate of A exact if the input samples were not quantized.

d. Based on your answers to parts (a) and (b) what would your estimate for A be if N is 4000. Explain why the estimate is not becoming exact even for very large N.

e. If we change the sampling frequency to $1600 \times \pi/3$ Hz will our estimate for A become exact for large N? If so, explain why and also show that the estimate becomes exact using Matlab.