EE 567

Homework 6

Due Tuesday, October 9, 2018 at 6:40 p.m.

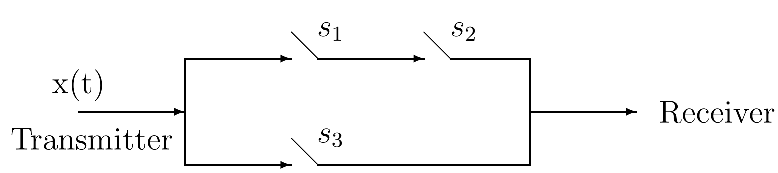
Work all 5 problems.

Problem 1.

a. Let W be a discrete random variable taking on values -2, 1, 3, 4 with $P(W = -2) = P(W = 1) = 1/10,\ P(W = 3) = 1/5,\ P(W = 4) = 3/5$. Find the expected value and variance of W.

b. Let X be a normally distributed random variable with mean 1 and variance 1. Suppose $Y = f(X)$, i.e., Y is a function of X. It is known that $E(Y) = 5$ and $Var(Y) = 36$. Furthermore, $r_{XY} = 1$, i.e., the correlation coefficient between X and Y is 1. Find the function f.

Problem 2. Consider the transmission of a signal as shown in the following diagram.



A signal is transmitted along two paths as shown. In the upper path there are two switches to pass through while in the lower path there is one switch to pass through. Each switch s_i operates independently and allows the signal to pass with probability p_i for $i = 1, 2, 3$. The signal transmission is successful if the signal $x(t)$ sent at the transmitter reaches the receiver along either or both paths. Find the probability that the transmission is successful if

a. $p_1 = 3/4,\ p_2 = 1/3,\ p_3 = 1/2$.

b. $p_1 = 3/4,\ p_2 = 1/3,\ p_3$ is a random variable with density

$$f(p) = \begin{cases} 2p, & 0 \le p \le 1 \\ 0, & \text{elsewhere.} \end{cases}$$

Problem 3. Consider the random process

$$X(t) = A\cos(\omega_c t + \theta)$$

where ω_c and θ are constants, A is a continuous random variable uniformly distributed on the interval $[0,\ A_{max}]$.

a. Find the mean function $\mu_X(t) = E[X(t)]$.

b. Find the correlation function $R_X(t_1, t_2)$ and determine if the process is wide sense stationary.

Problem 4. Consider a random process $X(t)$ defined by

$$X(t) = A\cos(2\pi f t) + B\cos(2\pi f t - \pi/2)$$

where f is a constant and A and B are random variables.

a. Show the condition
$$E[A] = E[B] = 0$$
is necessary for the random process to be stationary.

   Note that a random process is called stationary (or strict-sense stationary) if its distribution is insensitive to shifts in time t, that is, $X(t)$ and $X(t+T)$ have the same distribution function. If a process is stationary then it is also WSS and, in fact, all its moments are independent of t.

b. Show that $X(t)$ is wide sense stationary *if and only if* A and B are uncorrelated with equal variance σ^2.

c. Let $f = 1/2\pi$ and assume A and B are independent random variables and

$$P(A = -2) = P(B = -2) = 1/3, \quad P(A = 1) = P(B = 1) = 2/3.$$

   Show $X(t)$ is WSS but not strict-sense stationary. *Hint:* To show not strict-sense stationary compute $E[X^3(t)]$.

Problem 5. Use Matlab in parts of this problem.

Sometimes we use recursion formulas to compute something of interest. Consider the probability density function described by

$$f(x) = \begin{cases} ke^{-x}, & 0 \leq x \leq 1, \\ 0, & elsewhere \end{cases}$$

where k is a certain real number. Suppose we wish to compute the n^{th} moment $E_n = E[X^n]$, that is,

$$E_n = k \int_0^1 x^n e^{-x} dx.$$

a. Show that the value of k that makes $f(x)$ a valid density function is

$$k = \frac{1}{1 - e^{-1}}.$$

b. Using integration by parts show that

$$E_n = nE_{n-1} - ke^{-1}, \quad n = 1, 2, \ldots.$$

c. We can use this recursion formula to compute E_n for any desired n. Using the fact that $E_0 = 1$ use the above recursion formula in Matlab to compute E_9.

d. Now suppose that instead of using the value of k as given above you use $\hat{k}$ where

$$\hat{k} = 1.581977.$$

Note that $\hat{k}$ is equal to k to 6 decimal places. Now apply the recursion formula from part (b) using $\hat{k}$ instead of k and recompute E_9. Explain why your answer is nonsensical.

e. Let us rewrite our recursion formula as

$$E_{n-1} = \frac{E_n + ke^{-1}}{n}.$$

Now starting with E_{20} use this formula to compute E_9 again using $\hat{k} = 1.581977$. You will need a value to use for E_{20} to start the recursion (not the actual value of E_{20} but a rough approximation). Show that

$$E_n \leq \frac{k}{n+1}$$

and thus use 0 for E_{20} to start the recursion. Explain why your answer to E_9 here is much better than that obtained in part (d).