EE 567
Homework 4 solution

Problem 1. Over the interval 0 ≤ t ≤ 1 a PM signal is given by

$$s_{PM}(t) = 10cos2\pi f_0 t$$

where $f_0 = 100\ kHz.$ It is known that the carrier frequency is 75 kHz. If k_p = *1250* determine m(t) over the interval 0 ≤ t ≤ 1. (20 points)

Solution:
An PM signal has phase $\theta(t)$ linear to the modulating signal m(t), i.e.

$$\theta(t) \ = \ 2\pi f_c t \ + \ k_p m(t)$$

Hence, we have $s_{PM}(t) = \mathrm{A_c cos(2\pi f_c t\ +\ k_p m(t))}\ = 10cos2\pi f_0 t$, which then
gives us

$$k_p m(t) \ = \ 2\pi(f_0 - f_c)t$$

$$m(t) = \frac{2\pi \cdot 25 \cdot 10^3}{1250} = 40\pi t, t \in [0,1]$$

Problem 2. Over the interval 0 ≤ t ≤ 1 an FM signal is given by

$$s_{FM}(t) = 10cos2\pi f_0 t$$

where $f_0 = 100\ kHz.$ It is known that the carrier frequency is 75 kHz. If k_f = *1250* determine m(t) over the interval 0 ≤ t ≤ 1. (20 points)

Solution:
An FM signal has instantaneous frequency linear to the modulating signal m(t), i.e.

$$f_i(t) = f_c + \ k_f m(t)$$

The angle of the FM signal is thus $\theta(t) = \ 2\pi f_c t + 2\pi k_f \int_0^t m(s)ds.$
Hence, we have

$$s_{PM}(t) = \mathrm{A_c cos(2\pi f_c t}\ +\ 2\pi k_f \int_0^t m(s)ds)\ = 10cos2\pi f_0 t$$

$$\int_0^t m(s)ds \ = \frac{2\pi(f_0 - f_c)t}{2\pi k_f} = 20t$$

$$m(t) = 20, t \in [0,1]$$

Problem 3. An angle modulated signal is described by

$$s(t) = 10\,cos\,(2\pi f_c t + 0.1\,sin(2\pi f_1 t))$$

where f_c = *1 MHz* and f_1 = *1 kHz*.

a. Find the power of the modulated signal *s(t).* (10 points)

b. Find the frequency deviation, *Δf*. (10 points)

Solution:

a. Since the PM signal s(t) has amplitude 10, the power of the s(t) is

$$\mathrm{P} = \frac{10^2}{2} = 50$$

Note: The power of FM/PM signal does not change with different modulating signal m(t).

b. To find the frequency deviation *Δf*, we need to find its instantaneous frequency.

$$\theta(t) = 2\pi f_c t + 0.1 sin(2\pi f_1 t)$$

$$\mathrm{f_i}(t) = \frac{1}{2\pi}\frac{d\theta(t)}{dt} = f_c + \frac{f_1}{10} cos(2\pi f_1 t)$$

$$\Delta\mathrm{f} = \frac{1000}{10} = 100\ \mathrm{Hz}$$

Problem 4. A carrier wave of frequency 40 MHz is frequency-modulated by a sine-wave of amplitude 8 volts and frequency 15 kHz. The frequency sensitivity of the modulator is 10 kHz per volt.

a. Determine the approximate bandwidth of the FM wave using Carson's rule. (6 points)

b. Repeat part (a) assuming that the amplitude of the modulating wave is doubled. (7 points)

c. Repeat part (a) assuming that the modulation frequency is doubled. (7 points)

Solution:

Let the modulating signal be $m(t) = 10\,cos(2\pi f_m t)$, where $f_m = 1.5 \times 10^4$ Hz and $f_i(t) = f_c + k_f m(t)$, where $f_c = 4 \times 10^7$ Hz and $k_f = 10^4 Hz/V$.

$$\theta(t) = 2\pi f_c t + 2\pi k_f \int_0^t m(s)ds = 2\pi f_c t + \frac{8k_f}{f_m}\sin(2\pi f_m t)$$

Hence,

$$\mathrm{s(t)} = \mathrm{A_c}\cos(2\pi f_c t + \frac{8k_f}{f_m}\sin(2\pi f_m t))$$

where the modulation index $\beta = \frac{8k_f}{f_m} = \frac{16}{3}$

a, Carson's rule tells us that the effective bandwidth of the FM signal is approximately $B_{FM} = 2f_m(1+\beta)$. Thus,

$$\mathrm{B_{FM}} = 2 \times 1.5 \times 10^4 \times \frac{19}{3} = 1.9 \times 10^5\ Hz$$

b,

$$\mathrm{B_{FM}} = 2 \times 1.5 \times 10^4 \times \frac{3 + 2 \times 16}{3} = 3.5 \times 10^5\ Hz$$

c,

$$\mathrm{B_{FM}} = 2 \times 3 \times 10^4 \times \frac{3 + 16/2}{3} = 2.2 \times 10^5\ Hz$$

Problem 5. A certain AM signal is given as

$$s_{AM}(t) = [2 + cos(2\pi f_m t)]\ cos(2\pi f_c t).$$

The value of f_c is much greater than the bandwidth of the signal.

a. What is the modulating signal, m(t)? (6 points)
b. What is the modulation index? (7 points)
c. Determine the average message power in m(t). (7 points)

Solution:

a. The modulating signal is $m(t) = cos(2\pi f_m t)$

b. According to $s_{AM}(t)$, $\mathrm{A_c} = 2$ and $\mathrm{A_M} = 1$. Hence,

$$\mu = \frac{A_M}{A_c} = 0.5$$

c.

$$\mathrm{P_{mt}} = \frac{1}{2}$$