EE 567
Homework 3 solution

Problem 1.
Suppose

$$m(t) = \ sinc(100t), sinc(x) = \frac{\sin(x)}{x}$$

We form the DSB-SC signal as

$$s(t) \ = \ m(t)\ cos(10000\pi t).$$

Find S(f). (20 points)

Solution:
Using the Fourier transform pair $rect(\frac{t}{a}) \ \rightarrow \ asinc(a\pi f)$ and the duality property, we have

$$M(f) = \frac{1}{100} rect(\frac{f}{100})$$

The double-sideband suppressed carrier (DSB-SC) signal $s(t) = m(t)cos(10000\pi t)$ has Fourier transform

$$S(f) = \frac{1}{2}[M(f-5000) + M(f+5000)]$$
$$= \frac{1}{200}[rect\left(\frac{f-5000}{100}\right) + rect\left(\frac{f+5000}{100}\right)]$$

Problem 2.
Suppose

$$m(t) = \frac{1}{1+t^2}$$

We form the DSB-SC signal as

$$s(t) \ = \ m(t)\ cos(10000\pi t).$$

Find S(f). (20 points)

Solution:
Using the Fourier transform pair $e^{-a|t|} \ \rightarrow \frac{2a}{a^2+(2\pi f)^2}$ and the duality property, we have

$$M(f) = \pi e^{-2\pi|f|}$$

The double-sideband suppressed carrier (DSB-SC) signal has Fourier transform

$$S(f) = \frac{1}{2}[\pi e^{-2\pi|f-5000|} + \pi e^{-2\pi|f+5000|}]$$

Problem 3.
Suppose

$$m(t) = cos(100\pi t) + 2\,cos(300\pi t)$$

We form the DSB-SC signal as

$$s(t) \;=\; 2m(t)\,cos(1000\pi t).$$

Find S(f). (20 points)

Solution:

$$M(f) = \frac{1}{2}[\delta(f-50) + \delta(f+50)] + \delta(f-150) + \delta(f+150)$$

$$S(f) = M(f-500) + M(f+500)$$

Hence, its DSB-SC signal $s(t) \;=\; 2m(t)\,cos(1000\pi t)$ has Fourier transform

$$S(f) = \frac{1}{2}[\delta(f+450) + \delta(f+550)] + \delta(f+350) + \delta(f+650)$$
$$+ \frac{1}{2}[\delta(f-450) + \delta(f-550)] + \delta(f-350) + \delta(f-650)$$

Problem 4.
Suppose

$$m(t) = sin(100\pi t) sin(500\pi t)$$

We form the DSB-SC signal as

$$s(t) \;=\; 2m(t)\,cos(10000\pi t).$$

Find S(f). (20 points)

Solution:

$$M(f) = \frac{1}{4}[\delta(f-200) + \delta(f+200) - \delta(f-300) - \delta(f+300)]$$

Hence, its DSB-SC signal $s(t) \;=\; 2m(t)\,cos(10000\pi t)$ has Fourier transform

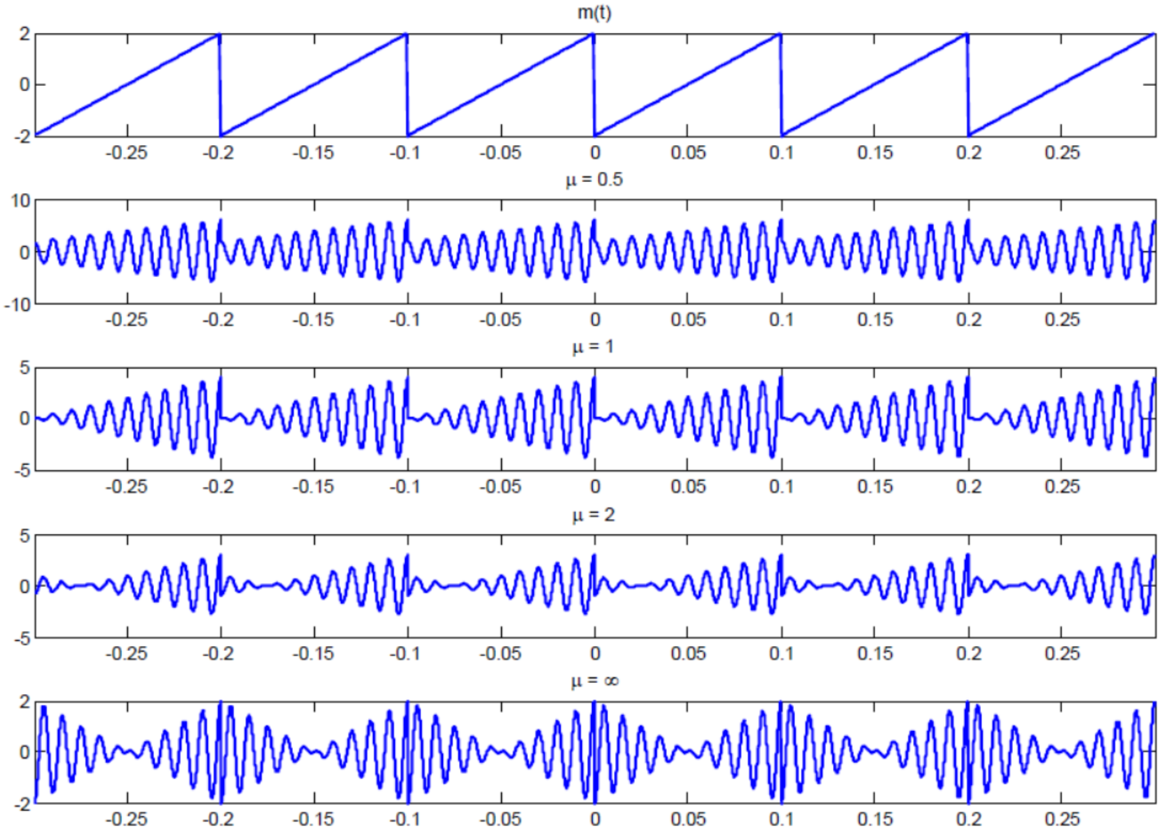
$$S(f) = \frac{1}{4}[\delta(f+4800) + \delta(f+5200) - \delta(f+4700) - \delta(f+5300)$$
$$+ \delta(f-4800) + \delta(f-5200) - \delta(f-4700) - \delta(f-5300)]$$

Problem 5.
Let

$$m(t) \;=\; \sum_{k=-\infty}^{\infty} (40(t-0.1k)-2)\,(u(t-0.1k) - u(t-0.1k-0.1))$$

Plot $s(t) = [A + m(t)]\cos(2\pi f_c t)$ for modulation indexes $\mu \;=\; 0.5, 1, 2, \infty$. Observe that $\mu \;=\; \infty$ is equivalent to DSB-SC modulation. Here t is in units of seconds and $f_c \;=\; 10$ Hz. (20 points)

Solution:



The signal m(t) and AM signals with modulation index $\mu = 0.5, 1, 2, \infty$.

Problem 6.

In this problem you can use Matlab but work as much analytically as you can. Suppose you transmit to someone the message signal

$$m(t) = \begin{cases} 1, & 0 \le t \le 1msec \\ 0, & elsewhere \end{cases}$$

The actual transmission is accomplished by using DSB-SC via

$$s(t) = m(t)\, cos(2\pi f_c t)$$

where, fc = 1 MHz. This is a finite duration signal so it has an infinite nonzero frequency content. At the receiver a bandpass filter is applied to s(t) so necessarily m(t) cannot be recovered perfectly in an actual system. Suppose an ideal bandpass filter $H_B(f)$ is used with height unity centered at $\pm f_c$ that extends from $f_c - B$ to $f_c + B$ and $-f_c - B$ to $f_c + B$. Let us define a distortion measure as

$$\mathrm{D_B} = 10\log_{10}\frac{\int_{-\infty}^{\infty}|S(f)H(f)|^2 df}{\int_{-\infty}^{\infty}|S(f)|^2 df}\,(\mathrm{dB})$$

so that no distortion $(\mathrm{B} = \infty)$ corresponds to $\mathrm{D_B} = 0$ dB.

a. Plot $\mathrm{D_B}$ for B = 200 Hz up to B = 20 KHz. (15 points)

b. Determine the smallest value of B such that DB > −1 dB. (5 points)

Solution:

Notice that the Fourier transform of m(t) is

$$\mathrm{M(f)} = \frac{1}{1000} sinc(\frac{f}{1000}) \exp(-j2\pi f \times 5 \times 10^{-4})$$

where s $sinc(x) = \frac{\sin(\pi x)}{\pi x}$. Suppose that an ideal bandpass filter is applied on $s(t)$, the distortion measure can be simplified as

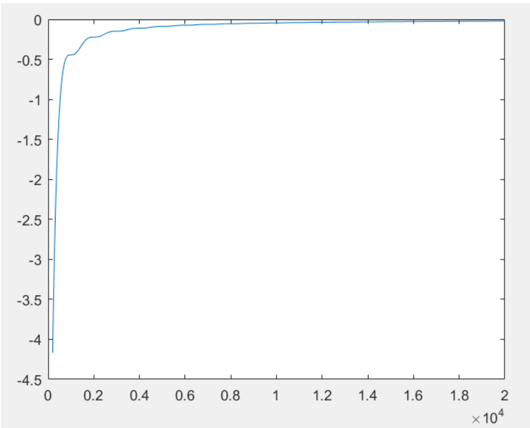
$$\mathrm{D} = 10 \log_{10} \frac{\int_{-\infty}^{\infty} |S(f)H(f)|^2 df}{\int_{-\infty}^{\infty} |S(f)|^2 df}$$

$$\mathrm{D} = 10 \log_{10} \frac{\int_{-B}^{B} |M(f)|^2 df}{\int_{-\infty}^{\infty} |M(f)|^2 df}$$

$$\mathrm{D} = 10 \log_{10}(10^3 \int_{-B}^{B} |M(f)|^2 df)$$

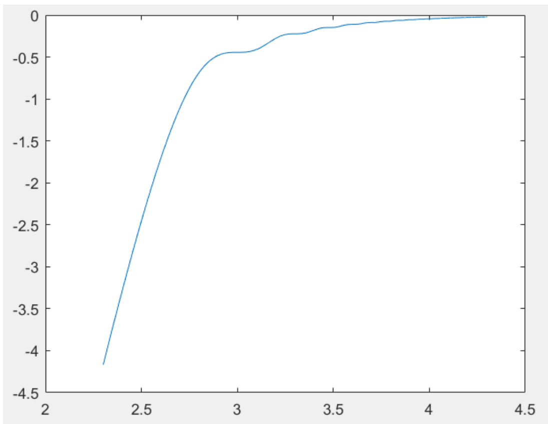
since Parseval's theorem gives $\int_{-\infty}^{\infty} |M(f)|^2 df = 10^{-3}$

The smallest bandwidth B that yields $\mathrm{D} > -1$ dB is around 527 Hz. (Other results greater than 526Hz, e.g., 530 Hz, will also be considered as correct).



f

or



Log(f)

Problem 7.

What does the concept of negative frequency mean? (20 points)

Solution:

The sign of frequency indicates the phase change direction. Assuming the positive frequency means the phase of the given frequency component increases with time, then the negative frequency means the phase of the given frequency component decreases with time.

Note: Any explanation that make sense could get the full points.