EE 567
Homework 2 solution

Problem 1. Let $g(t) = c_1 cos(2\pi f_c t\ +\ \theta_1) + c_1 cos(2\pi f_c t + \theta_2), c_1, c_1$, t are real numbers, $\theta_1 = \theta_2$.
Compute the time average message power in g(t). (20 points)

Solution:
The average power of g(t) is

$$P = \lim_{T\to\infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} |g(t)|^2\, dt$$

$$P = \lim_{T\to\infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} c_1^2\, cos^2(2\pi f_c t\ +\ \theta_1) + c_2^2\, cos^2(2\pi f_c t\ +\ \theta_2) + 2c_1 c_2 cos(2\pi f_c t\ +\ \theta_2) cos(2\pi f_c t\ +\ \theta_1)\, dt$$

$$P = \lim_{T\to\infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} \frac{c_1^2(1+cos(4\pi f_c t + 2\theta_1))}{2} + \frac{c_2^2(1+cos(4\pi f_c t + 2\theta_2))}{2} + c_1 c_2 [cos(4\pi f_c t\ +\ \theta_2 +\ \theta_1) + cos(\ \theta_2 -\ \theta_1))\, dt$$

$$P = \frac{c_1^2}{2} + \frac{c_2^2}{2} + c_1 c_2$$

Problem 2. Determine the time average message power and the rms value for each of the following signals:

a. $Asin(5t + \frac{\pi}{4})$ (4 points)

b. $Asin(5t + \frac{\pi}{4}) + Bsin(50t\ + \frac{\pi}{6})$ (8 points)

c. $Asin(3t)cos(6t)$ (8 points)

Solution:
Note that for a periodic signal g(t) with period T, the power of g(t) is $P = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} |g(t)|^2$and the root-mean-square (rms) value of g(t) is $rms = \sqrt{P}$

a. $\mathrm{P} = \lim_{\mathrm{T}\to\infty} \frac{1}{\mathrm{T}} \int_{-\frac{\mathrm{T}}{2}}^{\frac{\mathrm{T}}{2}} |\mathrm{g(t)}|^2\, \mathrm{dt} = \lim_{\mathrm{T}\to\infty} \frac{1}{\mathrm{T}} \int_{-\frac{\mathrm{T}}{2}}^{\frac{\mathrm{T}}{2}} \frac{\mathrm{A}^2 - \cos\left(10\mathrm{t}+\frac{\pi}{2}\right)}{2} \mathrm{dt} = \frac{\mathrm{A}^2}{2}$

$\mathrm{rms} = \frac{\mathrm{A}}{\sqrt{2}}$

b. $\mathrm{P} = \lim_{\mathrm{T}\to\infty} \frac{1}{\mathrm{T}} \int_{-\frac{\mathrm{T}}{2}}^{\frac{\mathrm{T}}{2}} \mathrm{A}^2 \sin^2\left(5\mathrm{t} + \frac{\pi}{4}\right) + \mathrm{B}^2 \sin^2\left(50\mathrm{t} + \frac{\pi}{6}\right) + 2\mathrm{AB}\sin\left(5\mathrm{t} + \frac{\pi}{4}\right) \sin\left(50\mathrm{t} + \frac{\pi}{6}\right) \mathrm{dt} = \frac{\mathrm{A}^2+\mathrm{B}^2}{2}$

$\text{rms} = \sqrt{\frac{A^2+B^2}{2}}$

c. $Asin(3t)\,cos(6t) = \frac{A}{2}sin(9t) - \frac{A}{2}sin(3t)$

Similar to 2. b

$P = (\frac{A^2}{4} + \frac{A^2}{4})/2 = \frac{A^2}{4}$

$\text{rms} = \frac{A}{2}$

Problem 3. Compute

a. $\int_{-2}^{2}(t^2+1)\delta(t-1)dt$ (10 points)

b. $\int_{-\infty}^{\infty}\cos(\frac{\pi}{2}(x-5))\,\delta(3x-3)dx$ (10 points)

Solution:

a. $\int_{-2}^{2}(t^2+1)\delta(t-1)dt = t^2+1|_{t=1} = 2$

b. b. $\int_{-\infty}^{\infty}\cos\left(\frac{\pi}{2}(x-5)\right)\delta(3x-3)dx$

$= \int_{-\infty}^{\infty}\frac{1}{3}\cos\left(\frac{\pi}{2}\left(\frac{t}{3}-5\right)\right)\delta(t-3)dt = \frac{1}{3}\cos\left(\frac{\pi}{2}\left(\frac{3}{3}-5\right)\right) = \frac{1}{3}$

Problem 4. Pulse Coded Modulation (PCM) is to be used to encode a signal. The signal ranges between the values -3 and +3. There are 4 bits or 16 levels (hence 16 code numbers) available. The levels assigned have symmetry like we demonstrated in class. The first three sample values obtained (before quantization) are 1.1, 2.7, and -2.7, respectively.

a. Find the quantized values for the three sample values. (10 points)

b. Find the corresponding PCM sequences for the quantized values. (10 points)

Solution:

a. Given the sample values 1.1, 2.7, and -2.7, we have quantized values 0.9375, 2.8125, and -2.8125.
b. The corresponding PCM sequences are 1010, 1111, and 0000.

Bin Interval	Quantized Value	PCM Codeword
[-3, -2.625)	-2.8125	0000
[-2.625, -2.25)	-2.4375	0001
[-2.25, -1.875)	-2.0625	0010
[-1.875, -1.5)	-1.6875	0011
[-1.5, -1.125)	-1.3125	0100
[-1.125, -0.75)	-0.9375	0101
[-0.75, -0.375)	-0.5625	0110
[-0.375, 0)	-0.1875	0111
[0, 0.375)	0.1875	1000
[0.375, 0.75)	0.5625	1001
[0.75, 1.125)	0.9375	1010
[1.125, 1.5)	1.3125	1011
[1.5, 1.875)	1.6875	1100
[1.875, 2.25)	2.0625	1101
[2.25, 2.625)	2.4375	1110
[2.625, 3]	2.8125	1111

Table 1: PCM Table

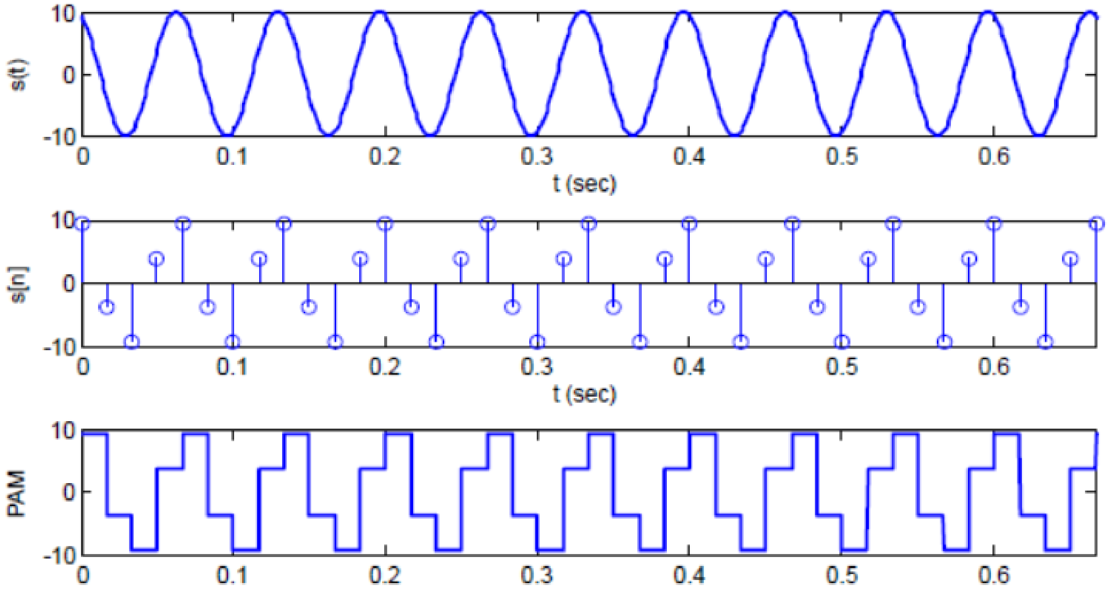
Problem 5. Let $s(t) = 10cos(2\pi ft + \frac{\pi}{8})$ where *f = 15 Hz*. Let us sample *s(t)* at the sampling rate of $fs = 60\ Hz$ to obtain the discrete time signal $s(nT_s) = 10cos(2\pi fnT_s + \frac{\pi}{8})$ where $T_s = 1/f_s$, for *n = 0, 1, 2, . . . , 40*. Using the PCM example in class as a guide compute the quantized PAM signal and the corresponding PCM codeword assuming you have 9 bits or 512 levels to represent the quantized signal. (20 points)

Note: In this problem you are to use Matlab. You should include your Matlab code with your homework submission.

Solution:

The sampling rate fs = 60Hz and we use 9 bits (512 levels) to represent the signal, we observe that s[n] have four values, 9.2388, -3.8268, -9.2388 and 3.8268, which belong to code number 20 within interval [9.2578, 9.2188), code number 354 within interval [-3.7891,- 3.8281], code number 493 with interval [-9.2188, -9.2578], and code number 159 with interval [3.7891, 3.8281] respectively.

Hence the PCM sequences are: 000010011, 101100001, 111101100, and 010011110


s(t)
t (sec)
s[n]
t (sec)
PAM