EE 567

Homework 1

Due Tuesday, September 4, 2018 at 6:40 p.m.

Work all 5 problems.

Problem 1. Compute the Fourier transform of $R_2(t)$ defined by

$$R_2(t) = \begin{cases} 1, & -2 \leq t \leq -1, \\ 1, & 1 \leq t \leq 2, \\ 0, & elsewhere \end{cases}$$

and simplify your answer to terms involving sinusoidal functions (i.e., without complex exponentials). Show all your work. Sketch a plot of its graph in the frequency domain. You should produce two plots (one for the magnitude of the Fourier transform and one for the phase). You may use Matlab to produce the plots.

Hint: It is quicker to utilize Fourier transform properties.

Problem 2. Determine which of the following systems is linear.

a. $y(t) = x(t) + 1$.

b. $y(t) = A \sin x(t)$, A is a constant.

c. $y(t) = x(at)$, a is a constant.

d. $y(t) = x(t - 1)$.

Problem 3. In class we sometimes move the limit operation inside or outside the integral. This problem illustrates this is not always valid (there are theorems in integration theory that specify when this is valid). Let

$$f_n(t) = nt(1 - t^2)^n, \quad 0 \le t \le 1.$$

a. Show $\lim_{n \to \infty} f_n(t) = 0$.

b. Show $\int_0^1 f_n(t)dt = \frac{n}{2(n+1)}$.

c. Deduce $\lim_{n \to \infty} \int_0^1 f_n(t)dt \neq \int_0^1 \lim_{n \to \infty} f_n(t)dt$.

Problem 4. Compute the Fourier transform of $x(t)$ where

$$x(t) = \frac{1}{\sqrt{2\pi}} \exp(-t^2/2).$$

Problem 5. As stated in class one of the Dirichlet conditions for the existence of the Fourier series for a function $x(t)$ is that the number of maxima and minima of $x(t)$ in a period is finite. For the Fourier transform this condition is replaced by the number of maxima and minima is finite in any finite interval. Give an example of a function that is continuous in the interval (0,1) but has an infinite number of maxima and minima in that interval and thus violates this condition. Prove that your function does indeed have an infinite number of maxima and minima in (0,1) and plot your function using Matlab illustrating this phenomena (as best you reasonably can with the plot).