EE 567 Final Exam Part 2 Solution

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Inst: Dr. C.W. Walker

Problem	Points	Score
4	16	
5	22	
6	12	
Part 2	50	
Part 1	50	
Parts 1+2	100	

Instructions and Information:

1) Print your name and indicate on-campus or DEN student at the top of the page.

2) Make sure this part of the exam has 3 problems.

3) This is a closed book exam. You may use two 8.5x11 inch sheet of notes (front and back). You may use a self-contained calculator but not a computer. Cell phones or any device with internet capability is not permitted. **You have 55 minutes to take this part of the exam**.

4) The points for each problem is shown above.

Problem 4. A binary communication system is used where at the receiver the test statistic is $z(T) = \pm A + n_0$, where $\pm A$ are equally likely, $A > 0$. The noise n_0 is distributed as

$$p(n_0) = \begin{cases} \frac{1}{2\beta} e^{-|n_0|/\beta}, \ -\infty < n_0 < \infty \\ \\ 0 \text{ elsewhere} \end{cases}$$

where $\beta = 0.1$. The user needs to estimate the sign of A to decide on the symbol sent. Determine the smallest value of A that yields a probability of decision error equal to 10^{-4}.

Solution: Assume WLOG that $+A$ is transmitted. Then the density of $Z = z(T)$ is

$$p_Z(z) = \frac{1}{2\beta} e^{-|z-A|/\beta}, \ -\infty < z < \infty.$$

The probability of decision error is then

$$\begin{aligned} P_E &= \int_{-\infty}^{0} p_Z(z) dz \\ &= \int_{-\infty}^{0} \frac{1}{2\beta} e^{(z-A)/\beta} dz = 10^{-4} \\ &\Rightarrow \ A = -\beta \ln(2 \times 10^{-4}) = 0.8517. \end{aligned}$$

Problem 5. Assume you have been hired to design a communication signaling system. There are 8 users all transmitting their data to a common receiver. The users are not coordinated with each other. Each user tries to transit data to the receiver when the data is available. Each user's data is composed of packets. Each packet has information bits only and no coding is applied (and there is no header). Each bit time is 1 msec.

At the receiver if two or more packets collide then all the data in each of the affected packets is considered lost. You can assume that the transmit time from each user to the receiver is the same.

You can assume Users 2 thru 8 will each independently start and end the attempted transmission of a packet sometime between 0 and 100 seconds where the start time is uniformly distributed between 0 and L seconds where L is such that L plus the packet time, P, is 100 seconds (this assures that the end of a packet transmission will always occur on or before the 100 second boundary marker).

User 1 though is special since User 1 always starts transmitting a packet at time 0 in each 100-second interval.

Each user will only transmit one packet per 100 second interval.

We wish to design the system so that the expected throughput for User 1 is maximized.

a. Let T_h denote the throughput for User 1. Write down the mathematical expression for the expected value of the throughput for User 1 as a function of P.

Solution:

$$E[T_h] = P\left(\frac{100-2P}{100-P}\right)^7 \frac{\text{sec}}{\text{interval}} \cdot \frac{1000\text{ msec per sec}}{100\text{ sec per interval}} \cdot \frac{1\text{ bit}}{\text{msec}}$$

b. Find (mathematically) the value of the packet length, P, that maximizes your expression in part (a). P should be determined to the nearest msec.

Solution: Compute

$$\frac{dE[T_h]}{dP} = 0$$

which becomes

$$\left(\frac{100 - 2P}{100 - P}\right)^6 \left[\frac{-700P}{(100 - P)^2} + \frac{100 - 2P}{100 - P}\right] = 0.$$

To find the correct P we consider the term in brackets [ ] above to get

$$P = 10.208 \text{ sec} = 10208 \text{ msec}.$$

c. Using your results from parts (a) and (b) determine the actual maximum expected throughput for User 1 in units of bps.

Solution: Plugging the value of P from part (b) into part (a) we get

$$E[T_h] = 43.9 \text{ bps}.$$

Problem 6. Short answers (no derivations needed).

a. List one advantage of digital communications over analog communications.
   Solution: Allow for the use of error correction coding.

b. What advantage do we gain in transmitting communication signals at high frequencies instead of low frequencies?

   Solution: We can use a smaller antenna to achieve the same gain as a larger antenna at a lower frequency.

c. What are the two primary kinds of error that lead to pointing losses by an antenna?

   Solution: Knowing where the receiver is actually located and being able to physically point where desired.

d. In cascading filters in a receiver do you want to place most of the gain in the first filter or the last filter in the cascaded chain (or does it matter)? Why?

   Solution: We want most of the gain in the first stage because in the noise figure equation for a cascaded system the gain in the first stage appears in the denominator of all subsequent terms in equation so it has the most influence in limiting noise effects later in the processing.

e. Write down the formula for probability of bit error for BPSK modulation in AWGN.

   Solution: $P_b = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$.

f. When using a frequency hopping spread spectrum system why would you not want to dwell very long at a particular frequency before hopping to another frequency?

   Solution: We do not want to dwell at a particular frequency very long since doing so might allow a jammer to locate our signal in the spectrum and jam us more directly by concentrating jammer energy in our signal bandwidth.