EE 567 Final Exam Part 1 Solution

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Inst: Dr. C.W. Walker

Problem	Points	Score
1	14	
2	15	
3	21	
Part 1	50	

Instructions and Information:

1) Print your name and indicate on-campus or DEN student at the top of the page.

2) Make sure this part of the exam has 3 problems.

3) This is a closed book exam. You may use two 8.5x11 inch sheet of notes (front and back). You may use a self-contained calculator but not a computer. Cell phones or any device with internet capability is not permitted. **You have 55 minutes to take this part of the exam**.

4) The points for each problem is shown above.

Problem 1. A carrier wave of frequency 50 MHz is frequency-modulated by a sine-wave of amplitude 6 volts and frequency 20 kHz. The frequency sensitivity of the modulator is 10 kHz per volt.

a. Determine the approximate bandwidth of the FM wave using Carson's rule.

Solution:

$$m(t) = A_m \cos(2\pi f_m t)$$

where $A_m = 6$ and $f_m = 20,000$. Then

$$f_i(t) = f_c + k_f m(t)$$

where $f_c = 50 \times 10^6$ and $k_f = 10,000$. Thus,

$$\theta(t) = 2\pi f_c t + 2\pi k_f \int_0^t m(s)ds = 2\pi f_c t + \frac{6k_f}{f_m}\sin(2\pi f_m t).$$

Hence,

$$s(t) = A_c \cos\left(2\pi f_c t + \frac{6k_f}{f_m}\sin(2\pi f_m t\right)$$

and we see that the modulation index is

$$\beta = \frac{6k_f}{f_m} = 3.0.$$

Carson's rule then says the approximate bandwidth is

$$B_{FM} = 2f_m(1+\beta) = 160 \text{ kHz}.$$

b. Repeat part (a) assuming that the modulation frequency is doubled.

Solution: Now $f_m = 40$ kHz and

$$\beta = \frac{6k_f}{f_m} = 1.5$$

and thus

$$B_{FM} = 2f_m(1+\beta) = 200 \text{ kHz}.$$

Problem 2. A receiver front end has a noise figure of 8 dB and a gain of 60 dB and a bandwidth of 6 MHz. The input signal power is 10^{-11} W. The antenna temperature is 175 K. Find $T_e,\ T_s,\ N_{out},\ SNR_{in}$ and SNR_{out}. You may use $T_0 = 290$ K and Boltzmann's constant equals 1.38×10^{-23} J/K.

Solution: We find with F =noise figure, T_a = antenna temperature, G = gain, B_n = bandwidth, S_{in} = input power

$$\begin{aligned}
T_e &= (10^{F/10} - 1)T_0 = 1540\ K \\
T_s &= T_a + T_e = 1715\ K \\
N_{in} &= kT_aB_n = 1.45 \times 10^{-14} \\
N_{out} &= kT_sB_n \times 10^{G/10} = 1.42 \times 10^{-7} \\
SNR_{in} &= \frac{S_{in}}{N_{in}} = 690 = 28.4\ dB \\
S_{out} &= S_{in} \times 10^{G/10} = 1.00 \times 10^{-5} \\
SNR_{out} &= \frac{S_{out}}{N_{out}} = 70.4 = 18.5\ dB.
\end{aligned}$$

Problem 3. Suppose we have a QPSK constellation where each point is at a radius of r_1 from the origin. The performance of this system depends on the distance between the constellation points and is mostly determined by the distance between adjacent points, d. It is a simple application of the Pythagorean theorem to show that $r_1 = d/\sqrt{2}$. With this system we transmit 2 bits per constellation symbol. Suppose now we consider an 8PSK constellation where we transmit 3 bits per symbol. If we used the same radius for the points as was used for the QPSK system the points would be closer to each other in distance since we have 8 points on the ring instead of 4. Thus, the probability of a symbol decision error at the receiver is higher for this design. Therefore, if we wish to maintain the same probability of symbol error for the 8PSK system that we had for the QPSK system we have to increase the radius of the ring to some value r_2 so that the distance between adjacent constellation points for the 8PSK system is also d.

a. Show that indeed $r_1 = d/\sqrt{2}$ as stated above.

   Solution: The vectors extending from the origin to two adjacent QPSK points makes a 90^o angle at the origin so

$$d^2 = r_1^2 + r_2^2 \Rightarrow r_1 = \frac{d}{\sqrt{2}}.$$

b. Find the required value of the radius r_2 in terms of d.

   Solution: For 8PSK let θ denote the 45^o degree angle from one point to the next adjacent point as measured from the origin. Then

$$d = 2r_2 \sin(\theta/2) \Rightarrow r_2 = \frac{d}{2\sin(\theta/2)} = \frac{d}{\sqrt{2-\sqrt{2}}} = 1.3066 \cdot d.$$

c. How much more power is required for the 8PSK system with a radius of r_2 to achieve the same probability of symbol error as for the QPSK system.

   Solution: The ratios of energies is

$$E_{ratio} = \frac{r_2^2/2}{r_1^2/2} = \frac{1}{1-\sqrt{1/2}} = 3.4142 = 5.33 \text{ dB}.$$